Math - Calculus II <u>SERIES (Partial Sums)</u>

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \ldots \quad \text{infinite series}$$

$$S_1 = a_1 \; ; \; S_2 = a_1 + a_2 \; ; \; S_3 = a_1 + a_2 + a_3 \; ; \; \ldots$$

$$S_n = a_1 + a_2 + a_3 + \ldots + a_n$$

$$\left\{ S_1 , S_2 , S_3 , \ldots , S_n , \ldots \right\} = \text{sequence of partial sums}$$

<u>GEOMETRIC SERIES</u>

$$a + ar + ar^2 + ar^3 + \ldots$$

$$S_1 = a \; ; \; S_3 = a + ar + ar^2 \; ; \; \ldots \rightarrow S_n = a + ar + ar^2 + \ldots + ar^{n-1}$$

$$S_n = \frac{a\,(1-r^n)}{1-r} \quad \text{(in compressed form)}$$

$$S = \text{sum of the series} = \lim_{n\to\infty} S_n = S_\infty = \frac{a}{1-r} \quad \text{(assuming this limit exists)}$$

Otherwise, the series diverges (that is, the series has no sum).

<u>Geometric Series</u>

Find S_n and determine whether the series converges. If the series converges, find the sum of the series.

(1) $\displaystyle\sum_{k=1}^{\infty} \frac{2}{5^{k-1}}$ $\quad S_n = \frac{5}{2}\left[1 - \left(\frac{1}{5}\right)^n\right]$; convergent to $S_\infty = \frac{5}{2}$

(2) $\displaystyle\sum_{k=1}^{\infty} \frac{2^{k-1}}{4}$ $\quad S_n = -\frac{1}{4}\left[1 - (2)^n\right]$; divergent

(3) $\displaystyle\sum_{n=1}^{\infty} 4^{n-1}$ $\quad S_n = -\frac{1}{3}\left[1 - (4)^n\right]$; divergent

(4) $\displaystyle\sum_{n=1}^{\infty} \left(-\frac{3}{2}\right)^{n+1}$ $\quad S_n = \frac{9}{10}\left[1 - \left(-\frac{3}{2}\right)^n\right]$; divergent

(5) $\displaystyle\sum_{n=1}^{\infty} \frac{4^{n+2}}{7^{n+1}}$ $\quad S_n = \frac{64}{21}\left[1 - \left(\frac{4}{7}\right)^n\right]$; convergent to $S_\infty = \frac{64}{21}$

(6) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{e}{\pi}\right)^{k-1}$ $\qquad S_n = \frac{\pi}{\pi - e}\left[1 - \left(\frac{e}{\pi}\right)^n\right]$; convergent to $S_\infty = \frac{\pi}{\pi - e}$

(7) $\displaystyle\sum_{k=1}^{\infty} \left(-\frac{1}{2}\right)^{k}$ $\qquad S_n = -\frac{1}{3}\left[1 - \left(-\frac{1}{2}\right)^n\right]$; convergent to $S_\infty = -\frac{1}{3}$

(8) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^{n+1}$ $\qquad S_n = \frac{4}{3}\left[1 - \left(\frac{2}{3}\right)^n\right]$; convergent to $S_\infty = \frac{4}{3}$

(9) $\displaystyle\sum_{k=1}^{\infty} \left(-\frac{3}{4}\right)^{k-1}$ $\qquad S_n = \frac{4}{7}\left[1 - \left(-\frac{3}{4}\right)^n\right]$; convergent to $S_\infty = \frac{4}{7}$

(10) $\displaystyle\sum_{k=1}^{\infty} (-1)^{k-1}\frac{7}{6^{k-1}}$ $\qquad S_n = 6\left[1 - \left(-\frac{1}{6}\right)^n\right]$; convergent to $S_\infty = 6$

<u>TELESCOPING SERIES</u>

Find S_n and if possible, find the sum of the series

(1) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ $\qquad S_n = 1 - \frac{1}{n+1}$; convergent to $S_\infty = 1$

(2) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$ $\qquad S_n = \frac{1}{2}\left[1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2}\right]$; convergent to $S_\infty = \frac{3}{4}$

(3) $\displaystyle\sum_{k=1}^{\infty} \frac{6}{(3k+2)(3k-1)}$ $\qquad S_n = 1 - \frac{2}{3n+2}$; convergent to $S_\infty = 1$

(4) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)}$ $\qquad S_n = \frac{1}{2}\left[1 - \frac{1}{2n+1}\right]$; convergent to $S_\infty = \frac{1}{2}$

(5) $\displaystyle\sum_{n=1}^{\infty} [\sin(n) - \sin(n+1)]$

$S_n = \sin(1) - \sin(n+1)$; divergent (oscillation) , no sum

(6) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$ $\qquad S_n = \ln(1) - \ln(n+1) = -\ln(n+1)$; divergent , no sum

(7) $\sum_{k=1}^{\infty}\left(\frac{1}{k+3}-\frac{1}{k+4}\right)$ $S_n = \frac{1}{4} - \frac{1}{n+4}$; convergent to $S_\infty = \frac{1}{4}$

(8) $\sum_{k=1}^{\infty}\frac{1}{(k+1)(k+2)}$ $S_n = \frac{1}{2} - \frac{1}{n+2}$; convergent to $S_\infty = \frac{1}{2}$

(9) $\sum_{n=1}^{\infty}\frac{4}{(n+3)(n+5)}$ $S_n = \frac{2}{4} + \frac{2}{5} - \frac{2}{n+4} - \frac{2}{n+5}$; convergent to $S_\infty = \frac{9}{10}$

Express the repeating decimal as a fraction by writing the decimal as a geometric series.

(1) $0.\overline{4}$ converges to $\frac{4}{9}$; (2) $0.\overline{9}$ converges to 1

(3) $5.\overline{37}$ converges to $\frac{532}{99}$; (4) $0.\overline{159}$ converges to $\frac{159}{999}$

(5) $0.\overline{7821}$ converges to $\frac{79}{101}$; (6) $0.451\overline{14}$ converges to $\frac{44663}{99000}$