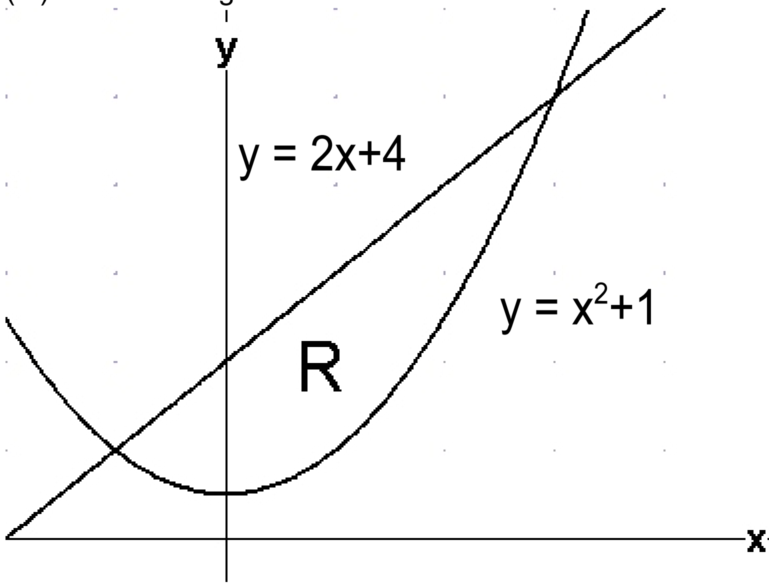
Math - Calculus II REVIEW # 6

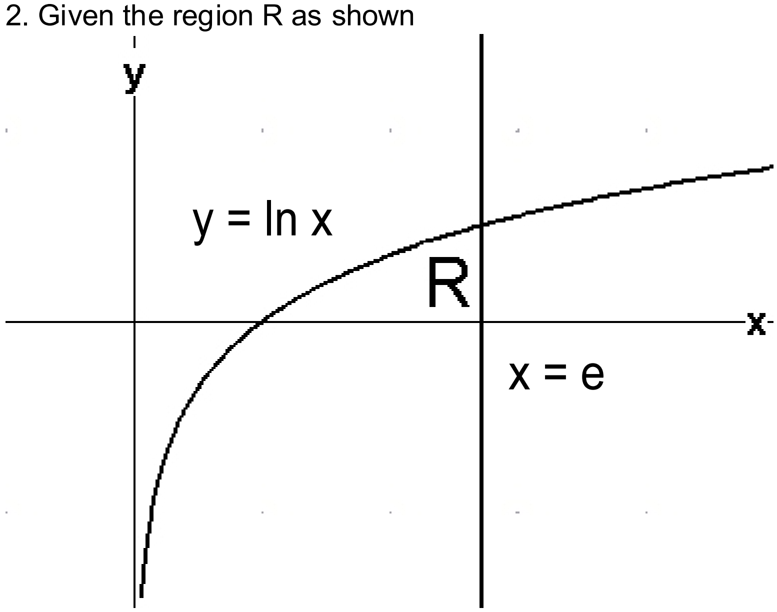
Areas and Volumes

(1) Given the region R as shown



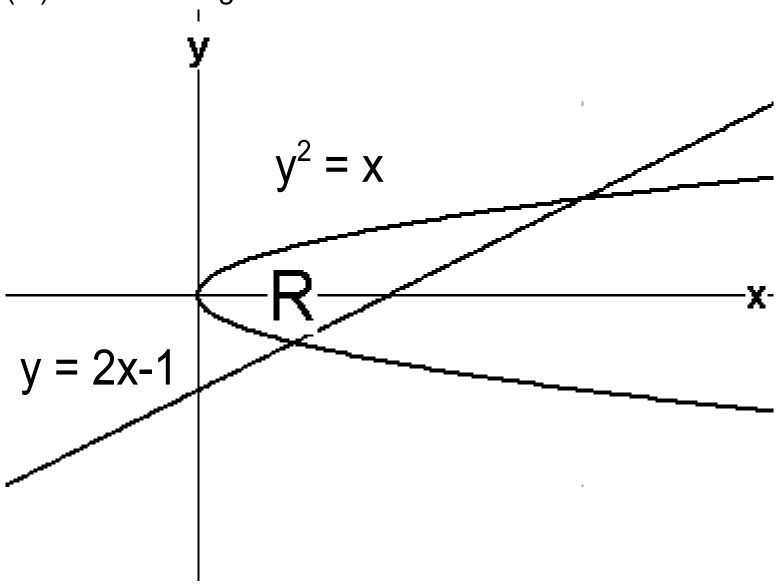
(a) find the points of intersection of the curves algebraically
(b) find the area of the region R
(c) find the volume of the solid of revolution when region R is revolved about the x-axis.
(d) find the volume of the solid of revolution when region R to the right of the y-axis is rotated about the y-axis.

2. Given the region R as shown



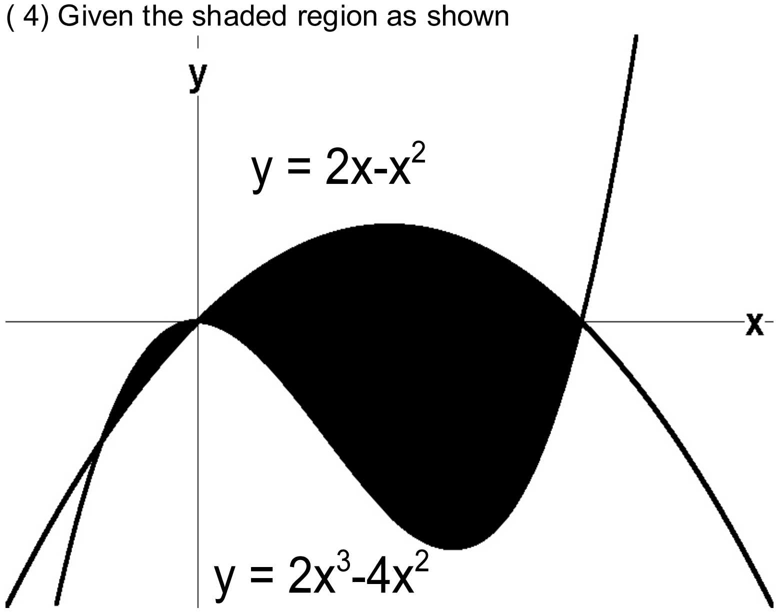
(a) find the points of intersection
(b) find the area of the region R
(c) find the volume of the solid of revolution when region R is revolved about the x-axis.
(d) find the volume of the solid of revolution when region R is revolved about the y-axis.

(3) Given the region R as shown



(a) find the points of intersection algebraically
(b) find the area of the region R

(4) Given the shaded region as shown



(a) find the points of intersection of the curves algebraically
(b) find the area of the shaded region
(c) set up the definite integral to find the volume of the solid generated when the shaded region to the left of the y-axis is revolved about the x-axis.
(d) set up the definite integral to find the volume of the solid generated when the shaded region to the right of the y-axis is rotated about the y-axis.

Answers:

(1 a) points : (3, 10) and (-1, 2) ; (1 b) $\dfrac{32}{3}$ square units ; (1 c) $\dfrac{1408\pi}{15}$ cubic units

(1 d) $\dfrac{45\pi}{2}$ cubic units

(2 a) points : (1, 0) ; (e, 0) ; (e, 1) ; (2 b) 1 square unit ; (2 c) $\pi(e-2)$ cubic units

(2 d) $\dfrac{(e^2+1)\pi}{2}$ cubic units

(3 a) points : (1, 1) ; $\left(\dfrac{1}{4}, -\dfrac{1}{2}\right)$; (3 b) $\dfrac{9}{16}$ square units

(4 a) points : $\left(-\dfrac{1}{2}, -\dfrac{5}{4}\right)$; (0, 0) ; (2, 0) ; (4 b) $\dfrac{131}{32}$ square units

Answers

(1 a) $x^2+1 = 2x+4 \Rightarrow x = -1\ ,\ x = 3$; pts: $(-1, 2)$; $(3, 10)$

(1 b) $A = \int_{-1}^{3} [(2x+4)-(x^2+1)]\ dx = \dfrac{32}{3}$ square units

(1 c) $V = \pi \int_{-1}^{3} [(2x+4)^2-(x^2+1)^2]\ dx = \dfrac{1408\pi}{15} \approx 93.87\pi$ cubic units (ring method)

(1 d) $V = 2\pi \int_{0}^{3} x(2x-x^2+3)\ dx = \dfrac{45\pi}{2}$ cubic units (shell)

(2 b) $A = \int_{1}^{e} \ln x\ dx = 1$ square unit

(2 c) $V = \pi \int_{1}^{e} (\ln x)^2\ dx = \pi(e-2)$ cubic units (disk) (parts twice)

(2 d) $V = 2\pi \int_{1}^{e} x \ln x\ dx = \dfrac{\pi}{2}(e^2+1)$ cubic units (shell) (parts)

(3) $A = \int_{-1/2}^{2} \left(\dfrac{1}{2}y+\dfrac{1}{2}-y^2\right)\ dy = \dfrac{9}{16}$ square units

(4 a) $2x^3-4x^2 = 2x-x^2 \Rightarrow x = -\dfrac{1}{2}\ ,\ 0\ ,\ 2$

(4 b) $A = \int_{-1/2}^{0} (2x^3-3x^2-2x)\ dx + \int_{0}^{2} (2x+3x^2-2x^3)\ dx = \dfrac{131}{32}$ square units

(4 c) $V = \pi \int_{-1/2}^{0} [(2x-x^2)^2-(2x^3-4x^2)^2]\ dx = \dfrac{5\pi}{56}$ cubic units (ring method)

(4 d) $V = 2\pi \int_{0}^{2} x[(2x-x^2)-(2x^3-4x^2)]\ dx = \dfrac{136\pi}{15}$ cubic units (shell)