Math - Calculus II REVIEW # 5

Give the method only to solve the following:

(1) $\int \sin^4\left(\frac{x}{2}\right) dx$; (2) $\int \cos^5 3x\ dx$; (3) $\int \csc^6 2x\ dx$

(4) $\int \tan^5\theta\ d\theta$; (5) $\int \csc^4 x \cot^2 x\ dx$; (6) $\int (\cot x)^{1/2} \csc^2 x\ dx$

(7) $\int \sin^3 x \cos^2 x\ dx$; (8) $\int \frac{\sec^2 x}{\tan^3 x} dx$; (9) $\int \frac{\tan x}{\sec^3 x} dx$

(10) $\int \frac{\cos^3 3x}{\sqrt[3]{\sin 3x}} dx$; (11) $\int \sin^4 2x \cot^3 2x\ dx$; (12) $\int \frac{\csc^4 x}{\cot^2 x} dx$

(13) $\int \frac{\sec x + \tan x}{\sec^2 x} dx$; (14) $\int (\sec^2 x + \csc^2 x)\ dx$

(15) $\int \frac{\sin^2 x}{1 - \cos x} dx$; (16) $\int \sin^2 \pi x \cos^2 \pi x\ dx$; (17) $\int \frac{\sin^2 \pi x}{\cos^6 \pi x} dx$

(18) $\int \frac{1}{\sin x \cos x} dx$; (19) $\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$; (20) $\int x^2 e^{-5x}\ dx$

(21) $\int (\ln x)^2\ dx$; (22) $\int \frac{(\ln x)^7}{x} dx$; (23) $\int \arcsin x\ dx$

(24) $\int \frac{dx}{x \ln x}$; (25) $\int \frac{e^{\tan y}}{\cos^2 y} dy$; (26) $\int \frac{x^2+1}{x\sqrt{x^2-1}} dx$

(27) $\int \sqrt{\sec y}\ \tan y\ dy$; (28) $\int \cos 3x \cos 7x\ dx$

(29) $\int (\tan\theta + \cot\theta)^2\ d\theta$; (30) $\int \frac{x\ dx}{\sqrt{4-x^2}}$; (31) $\int \frac{x^3\ dx}{\sqrt{4-x^2}}$

(32) $\int \frac{x+5}{\sqrt{1-x^2}} dx$; (33) $\int \frac{x^2+2x+3}{x^2+1} dx$; (34) $\int \frac{\arctan x}{x^2+1} dx$

(35) $\int \frac{\sqrt{\cot x}}{\sin^2 x}\,dx$; (36) $\int \csc^3 x\,dx$; (37) $\int \tan^2 x \sec x\,dx$

(38) $\int (x+1)^2 e^{-3x}\,dx$; (39) $\int \tan^4 x\,dx$; (40) $\int \frac{\ln(x+1)}{x+1}\,dx$

(41) $\int \ln(x+1)\,dx$; (42) $\int x^5 e^{x^3}\,dx$; (43) $\int \sqrt{1 - \sin x}\,dx$

Answers:

(1) double angle: $\int \left(\frac{(1 - \cos x)^2}{2}\right) dx = \frac{1}{4}\int (1 - 2\cos x + \cos^2 x)\,dx$

and repeat for $\int \cos^2 x\,dx$

(2) $\boxed{u = \sin 3x}$ $\int (1 - \sin^2 3x)^2 \cos 3x\,dx \rightarrow \frac{1}{3}\int (1-u^2)^2\,du$

(3) $\boxed{u = \cot 2x}$ $\int (1 + \cot^2 2x)^2 \csc^2 2x\,dx \rightarrow -\frac{1}{2}\int (1+u^2)^2\,du$

(4) $\boxed{u = \sec \theta}$ $\int \frac{(\sec^2 \theta - 1)^2}{\sec \theta} \sec\theta \tan\theta\,d\theta \rightarrow \int (u^2-1)^2 u^{-1}\,du$

(5) $\boxed{u = \cot x}$ $\int (1 + \cot^2 x) \cot^2 x \csc^2 x\,dx \rightarrow -\int (1+u^2)u^2\,du$

(6) $\boxed{u = \cot x}$ $-\int u^{1/2}\,du$

(7) $\boxed{u = \cos x}$ $\int (1 - \cos^2 x) \cos^2 x \sin x\,dx \rightarrow -\int (1-u^2)u^2\,du$

(8) $\boxed{u = \tan x}$ $\int u^{-3}\,du$

(9) $\boxed{u = \sec x}$ $\int (\sec x)^{-4} \sec x \tan x\,dx \rightarrow \int u^{-4}\,du$

(10) $\boxed{u = \sin 3x}$ $\int (\sin 3x)^{-1/3} (1 - \sin^2 3x) \cos 3x\,dx \rightarrow \frac{1}{3}\int u^{-1/3}(1-u^2)\,du$

(11) $\boxed{u = \cos 2x}$ $\int \sin^4 2x \dfrac{\cos^3 2x}{\sin^3 2x}\, dx \rightarrow -\dfrac{1}{2}\int u^3\, du$

(12) $\boxed{u = \cot x}$ $\int (\cot x)^{-2}\,(1 + \cot^2 x)\csc^2 x\, dx \rightarrow -\int u^{-2}(1+u^2)\, du$

(13) $\boxed{u = \sin x}$

$$\int \frac{1}{\sec x}\, dx + \int \frac{\tan x}{\sec^2 x}\, dx = \int \cos x\, dx + \int \sin x \cos x\, dx$$

$$\int \sin x \cos x\, dx \rightarrow \int u\, du$$

(14) use formulas

(15) factor and reduce: $\int \dfrac{1 - \cos^2 x}{1 - \cos x}\, dx = \int (1 + \cos x)\, dx = x + \sin x + C$

(16) double angle:

$$\int \left(\frac{1 - \cos 2\pi x}{2}\right)\left(\frac{1 + \cos 2\pi x}{2}\right) dx = \frac{1}{4}\int (1 - \cos^2 2\pi x)\, dx$$

$$\frac{1}{4}x - \frac{1}{4}\int \left(\frac{1 + \cos 4\pi x}{2}\right) = \frac{1}{4}x - \frac{1}{8}\int dx - \frac{1}{8}\int \cos 4\pi x\, dx$$

(17)

$\boxed{u = \tan \pi x}$ $\int \tan^2 \pi x \sec^4 \pi x\, dx = \int \tan^2 \pi x\,(1 + \tan^2 \pi x)\sec^2 \pi x\, dx$

$$\frac{1}{\pi}\int u^2(1+u^2)\, du$$

(18) $\int \dfrac{\sin^2 x + \cos^2 x}{\sin x \cos x}\, dx = \int \dfrac{\sin^2 x}{\sin x \cos x}\, dx + \int \dfrac{\cos^2 x}{\sin x \cos x}\, dx$

$\int \tan x\, dx + \int \cot x\, dx$; use formulas

(19) $\boxed{u = \sqrt{x}}$ $\rightarrow 2\int \sin u\, du$

(20) parts: $u = x^2$; $dv = e^{-5x}\, dx$; tabular form

(21) parts (twice): $u = (\ln x)^2 \ ; \ dv = 1 \ dx$

(22) $\boxed{u = \ln x}$ ➜ $\int u^7 \, du$

(23) parts: $u = \arcsin x \ ; \ dv = 1 \ dx$

(24) $\boxed{u = \ln x}$ ➜ $\int u^{-1} \, du$

(25) $\boxed{u = \tan y}$ ➜ $\int e^u \, du$

(26) split: $\int \frac{x^2}{x\sqrt{x^2-1}} \, dx + \int \frac{1}{x\sqrt{x^2-1}} \, dx = \int \frac{x}{\sqrt{x^2-1}} \, dx + \int \frac{1}{x\sqrt{x^2-1}} \, dx$

$\boxed{u = x^2-1}$ $\int \frac{x}{\sqrt{x^2-1}} \, dx$ ➜ $\frac{1}{2} \int u^{-1/2} \, du$; use formula: $\int \frac{1}{x\sqrt{x^2-1}} \, dx$

(27) $\boxed{u = \sec y}$ ➜ $\int u^{1/2} \, u^{-1} \, du = \int u^{-1/2} \, du$

(28) parts: (double back) $u = \cos 3x \ ; \ dv = \cos 7x \, dx$

(29) expand, reduce and use formula:

$$\int (\tan^2 \theta + 2 \tan \theta \cot \theta + \cot^2 \theta) \, d\theta$$

$$\int (\sec^2 \theta - 1 + 2 + \csc^2 \theta - 1) \, d\theta = \int (\sec^2 \theta + \csc^2 \theta) \, d\theta$$

(30) $\boxed{u = 4-x^2}$ ➜ $-\frac{1}{2} \int u^{-1/2} \, du$

(31) alg sub: $\boxed{u = 4-x^2}$ ➜ $-\frac{1}{2} \int u^{-1/2} (4-u) \, du$

(32) split: $\int \frac{x \, dx}{\sqrt{1-x^2}} + 5 \int \frac{dx}{\sqrt{1-x^2}}$

$\boxed{u = 1-x^2}$ $\int \frac{x \, dx}{\sqrt{1-x^2}} \, dx$ ➜ $-\frac{1}{2} \int u^{-1/2} \, du$; use formula: $5 \int \frac{dx}{\sqrt{1-x^2}}$

(33) long division, sub $\boxed{u = x^2+1}$ and formula:

$$\int \left(1 + \frac{2x+2}{x^2+1}\right) dx = \int 1\ dx + \int \frac{2x}{x^2+1}\ dx + 2\int \frac{dx}{x^2+1}$$

(34) $\boxed{u = \arctan x}$ ➜ $\int u\ du$

(35) $\boxed{u = \cot x}$ ➜ $-\int u^{1/2}\ du$

(36) parts (double back): $u = \csc x\ ;\ dv = \csc^2 x\ dx$

(37) identity, parts (double back) and formula:

$$\int (\sec^2 x - 1) \sec x\ dx = \int \sec^3 x\ dx - \int \sec x\ dx$$

$$\int \sec^3 x\ dx \ \text{➜}\ u = \sec x\ ;\ dv = \sec^2 x\ dx$$

(38) parts: $u = (x+1)^2\ ;\ dv = e^{-3x}\ dx$ (tabular form)

(39) identity and formula:

$$\int \tan^2 x\ (\sec^2 x - 1)\ dx = \int \tan^2 x \sec^2 x\ dx - \int (\sec^2 x - 1)\ dx$$

$\boxed{u = \tan x}$ $\int \tan^2 x \sec^2 x\ dx$ ➜ $\int u^2\ du$

(40) $\boxed{u = \ln (x+1)}$ ➜ $\int u\ du$

(41) parts: $u = \ln(x+1)\ ;\ dv = 1\ dx$ and long division

(42) $\boxed{t = x^3}$ ➜ $\frac{1}{3}\int t\ e^t\ dt$; then parts: $u = t\ ;\ dv = e^t\ dt$

(43) rationalize:

$$\int \frac{\sqrt{1 - \sin x}\ .\ \sqrt{1 + \sin x}}{\sqrt{1 + \sin x}}\ dx = \int \frac{\sqrt{1 - \sin^2 x}}{\sqrt{1 + \sin x}}\ dx = \int \frac{\cos x}{\sqrt{1 + \sin x}}\ dx$$

$\boxed{u = 1 + \sin x}$ ➜ $\int u^{-1/2}\ du$