Math - Calculus II EXPONENTIAL INTEGRALS

Rules for simplifying exponents:

$$\frac{a^m}{a^n} = a^{m-n} \quad ; \text{example:} \quad \frac{5^{3x}}{5^{4x}} = 5^{-x} = \frac{1}{5^x}$$

$$a^m \,.\, a^n = a^{m+n} \quad ; \text{example:} \quad 10^x \,.\, 10^{x^2} = 10^{x+x^2}$$

$$(a^m)^n = a^{mn} \quad ; \text{example:} \quad (5^x)^2 = 5^{2x} \;;\; (4^x)^{-x} = 4^{-x^2}$$

$$a^m \,.\, a^m = (ab)^m \quad ; \text{example:} \quad 4^x \,.\, 5^x = 20^x$$

Derivatives:

$$f(x) = e^u \rightarrow f'(x) = e^u\, u' \quad ; \text{example:} \quad y = e^{4x^2} \rightarrow y' = 8x\, e^{4x^2}$$

$$f(x) = a^u \rightarrow f'(x) = a^u \ln(a)\, u' \quad ; \text{example:} \quad y = 2^{\tan x} \rightarrow y' = 2^{\tan x} \sec^2 x \ln(2)$$

Integrals:

$$\int e^u\, du = e^u + C \quad ; \quad \int a^u\, du = \frac{a^u}{\ln(a)} + C$$

Determine:

(1) $\int e^x\, dx$; (2) $\int 4^x\, dx$; (3) $\int e^{-x}\, dx$; (4) $\int e^{4x}\, dx$

(5) $\int e^{-\frac{1}{2}x}\, dx$; (6) $\int e^{3x^2}\, x\, dx$; (7) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$; (8) $\int e^{4x} \,.\, e^{3x}\, dx$

(9) $\int \frac{e^{5x}}{e^{3x}}\, dx$; (10) $\int 4^{x^3}\, x^2\, dx$; (11) $\int 10^{-x}\, dx$; (12) $\int 7^{x^2}\, x\, dx$

(13) $\int \frac{5^{x^2}}{5^4}\, x\, dx$; (14) $\int \frac{x^2\, dx}{e^{4x^3}}$; (15) $\int \frac{5\, e^{\frac{3}{x}}}{x^2}\, dx$; (16) $\int \frac{3^{\sqrt[3]{2x+1}}}{\sqrt[3]{(2x+1)^2}}\, dx$

(17) $\int 4^{\sin x} \cos x\, dx$; (18) $\int \frac{e^{\tan x}}{\cos^2 x}\, dx$; (19) $\int \frac{e^{4 \sec x} \sin x}{\cos^2 x}\, dx$

(20) $\int 6^{-\frac{x}{4}}\, dx$

Answers:

(1) $e^{x} + C$; (2) $\dfrac{4^{x}}{\ln(4)} + C$; (3) $-e^{-x} + C$; (4) $\dfrac{1}{4}\, e^{4x} + C$

(5) $-2\, e^{-\frac{1}{2}x} + C$; (6) $\dfrac{1}{6}\, e^{3x^2} + C$; (7) $2\, e^{\sqrt{x}} + C$; (8) $\dfrac{1}{7}\, e^{7x} + C$

(9) $\dfrac{1}{2}\, e^{2x} + C$; (10) $\dfrac{4^{x^3}}{3\, \ln(4)} + C$; (11) $-\dfrac{10^{-x}}{\ln(10)} + C$

(12) $-\dfrac{7^{x^2}}{2\, \ln(7)} + C$; (13) $\dfrac{5^{x^2-4}}{2\, \ln(5)} + C$ or $\dfrac{5^{x^2}}{2\, (5^4)\, \ln(5)} + C$

(14) $-\dfrac{1}{12}\, e^{-4x^3} + C$; (15) $-\dfrac{5}{3}\, e^{\frac{3}{x}} + C$; (16) $\dfrac{3}{2}\, \dfrac{3^{\sqrt[3]{2x+1}}}{\ln(3)} + C$

(17) $\dfrac{4^{\sin x}}{\ln(4)} + C$; (18) $e^{\tan x} + C$; (19) $\dfrac{1}{4}\, e^{4 \sec x} + C$

(20) $-4\, \dfrac{6^{-\frac{x}{4}}}{\ln(6)} + C$