Math - Calculus II SEPARABLE DIFFERENTIAL EQUATIONS

Notation: y(a) = b means when x = a , y = b , in other words, the graph of y = f(x) passes through the point (a, b)

Solve:

(1) $\dfrac{dy}{dx} = 2x$; (2) $\dfrac{dy}{dx} = x^2 y$; (3) $\dfrac{dy}{dx} = \dfrac{1}{x}$; (4) $(y+1)\dfrac{dy}{dx} = 2x$

(5) $3y^2 \dfrac{dy}{dx} = 1$; (6) $(1+y)\dfrac{dy}{dx} - 4x = 0$; (7) $y' - xy = 0$

(8) $y' - y = 5$; (9) $\dfrac{dy}{dt} = \dfrac{e^t}{4y}$; (10) $\dfrac{dy}{dx} = \sqrt{\dfrac{x}{y}}$; (11) $e^y \dfrac{dy}{dt} = 3t^2+1$

(12) $\dfrac{dy}{dx} = \dfrac{x^2-2}{3y^2}$; (13) $(2+x)\, y' = 2y$; (14) $xy' = y$; (15) $\dfrac{dy}{dx} = \sqrt{1-y}$

(16) $y' - y\,(x-1) = 0$; (17) $y' = (2x-1)(y+3)$; (18) $e^x\,(y' + 1) = 1$

(19) $y' = \dfrac{x}{y} - \dfrac{x}{1+y}$; (20) $y\,y' - 2x\,e^x = 0$

(21) $y\,y' - e^x = 0$; $y(0) = 4$; (22) $\sqrt{x} + \sqrt{y}\, y' = 0$; $y(1) = 4$

(23) $x(y+4) + y' = 0$; $y(0) = -5$; (24) $dP - 6P\,dt = 0$; $P(0) = 5$

(25) $\dfrac{dy}{dx} = x^2\,(1+y)$; $y(0) = 3$

(26) $dT + k(T-70)\,dt = 0$; $T(0) = 140$

Answers:

(1) $y = x^2 + C$; (2) $\ln y = \frac{x^3}{3} + C_1 \rightarrow y = C\, e^{\frac{x^3}{3}}$; (3) $y = \ln x + C$

(4) $\frac{y^2}{2} + y = x^2 + C \rightarrow \frac{(y+1)^2}{2} = x^2 + C_1 \rightarrow (y+1)^2 = 2x^2 + C$

$\rightarrow y+1 = \pm\sqrt{2x^2 + C} \rightarrow y = \pm\sqrt{2x^2 + C} - 1$

(5) $y^3 = x + C \rightarrow y = \sqrt[3]{x+C}$

(6) $y + \frac{y^2}{2} = 2x^2 + C \rightarrow \frac{(1+y)^2}{2} = 2x^2 + C_1 \rightarrow y = \pm\sqrt{4x^2+C} - 1$

(7) $\ln y = \frac{x^2}{2} + C_1 \rightarrow y = C\, e^{\frac{x^2}{2}}$

(8) $\ln(y+5) = x + C_1 \rightarrow y+5 = C\, e^x \rightarrow y = C\, e^x - 5$

(9) $2y^2 = e^t + C \rightarrow y = \pm\sqrt{\frac{e^t+C}{2}}$

(10) $\frac{2}{3}y^{3/2} = \frac{2}{3}x^{3/2} + C_1 \rightarrow y^{3/2} = x^{3/2} + C \rightarrow y = (x^{3/2} + C)^{2/3}$

(11) $e^y = t^3+t+C \rightarrow y = \ln(t^3+t+C)$

(12) $y^3 = \frac{x^3}{3} - 2x + C \rightarrow y = \sqrt[3]{\frac{x^3}{3} - 2x + C}$

(13) $\ln y = 2\ln(2+x) + C_1 \rightarrow y = C\, e^{2\ln(x+2)} = C\, e^{\ln(x+2)^2} = C(x+2)^2$

(14) $\ln y = \ln x + \ln C = \ln(Cx) \rightarrow y = Cx$

(15) $-2(1-y)^{1/2} = x + C_1 \rightarrow (1-y)^{1/2} = -\frac{1}{2}x + C$

$$\Rightarrow \quad 1-y = \left(-\frac{1}{2}x + C\right)^2 \quad \Rightarrow \quad y = 1 + \left(-\frac{1}{2}x + C\right)^2$$

(16) $\ln y = \dfrac{x^2}{2} - x + C_1 \quad \Rightarrow \quad y = C\, e^{\frac{x^2}{2} - x}$

(17) $\ln(y+3) = x^2 - x + C_1 \quad \Rightarrow \quad y+3 = C\, e^{x^2-x} \quad \Rightarrow \quad y = C\, e^{x^2-x} - 3$

(18) $y = -e^{-x} - x + C$

(19) $\dfrac{y^2}{2} + \dfrac{y^3}{3} = \dfrac{x^2}{2} + C_1 \quad \Rightarrow \quad 3y^2 + 2y^3 = 3x^2 + C$

(20) $\dfrac{y^2}{2} = 2\,(x\,e^x - e^x) + C_1 \quad \Rightarrow \quad y^2 = 4(x\,e^x - x) + C$

$$\Rightarrow \quad y = \pm\sqrt{4(x\,e^x - x) + C}$$

(21) $\dfrac{y^2}{2} = e^x + C_1 \quad \Rightarrow \quad y^2 = 2e^x + C \quad \Rightarrow \quad y^2 = 2e^x + 14$

(22) $y^{3/2} = -x^{3/2} + 9$; (23) $y+4 = -e^{-\frac{x^2}{2}}$; (24) $P = 5\,e^{6t}$

(25) $y = 4e^{\frac{x^3}{3}} - 1$; (26) $T-70 = 70\,e^{-kt}$