Word Problems (Simplex)

1. The California Dried Fruit Company sells three different packages of dried fruit, Regular, Special and Deluxe. The amount of fruit (in kilograms) per package is as follows:

	Regular	Special	Deluxe
Apricots	1	2	2
Apple Slices	2	1	2
Raisins	1	1	1

The company has on hand $3,000$ kilograms of apricots, $3,500$ kilograms of apple slices and $4,000$ kilograms of raisins. To meet the demand, at least $1,200$ packages must be produced. If the profits are $\$2.25$, $\$2.50$ and $\$2.00$ for each package of Regular, Special and Deluxe, respectively, find the production required to maximize profit.

Define your variables, set up the objective function and all the constraints to find the maximum profit.

2. The Newtronic company produces plastic casings for modems, walkmans and cellular telephones, each requiring different amount of plastic, production time and packaging costs as shown below:

	Plastic	Production Time	Packaging Costs
Cellular phone	120 gm	20 min.	\$1.00
Walkman	90 gm	30 *min.*	\$1.50
Modem	170 gm	15 min.	\$1.25

Each week the company has a supply of 200 000 gm of plastic,operates 500 man-hours and has \$1 800 allotted for packaging costs. It also has contracted to supply at least 1 200 casings to the customers. If the profit for each cellular telephone, walkman and modem casing is \$6 , \$8 and \$11 respectively, find the number of units of each product to be produced for maximum profit.

Set Up to solve this problem:

(a) define all the variables used;

(b) state the objective and identify the objective function;

(c) state ALL the constraints.

3. The ABC Nut Company has a daily supply of 500 lbs. of cashews, 100 lbs. of pecans and 50 lbs. of peanuts. The company produces 3 types of products, each sold in cans containing 5 lbs. of nuts.

Products	Cashews	Pecans	Peanuts	Price/Can
I	2	1	2	\$4.25
II	3	1	1	\$5
III	4	$\frac{1}{2}$	$\frac{1}{2}$	\$6.50

How many cans of each type should be made on a daily basis to maximize revenue?

Set Up to solve this problem by:

(a) defining all the variables used.

(b) identifying the objective function;

(c) stating all the constraints.

You do not have to solve this problem!

4. A firm has decided to discontinue production of an unprofitable product. This will create excess capacity, and the firm is considering one or more of three possible new products, A, B, and C. The available weekly hours in the plant will be 477 hours in tool and die, 350 hours on the drill presses, and 150 hours on lathes. The hours of production required in each of these areas are as follows for each of the products.

	Tool and Die	Drill Press	Lathe
A	9	5	3
B	3	4	0
C	0.5	0	2

Furthermore, the sales department foresees no limitations on the sale of products A and C, but sees sales of only 20 or fewer per week for B. If the unit profits expected are \$30 for A, \$9 for B, and \$15 for C, how many of each should be produced to maximize profits? What is the maximum profit?
Set Up to solve this problem by:

(a) defining all the variables used.

(b) state the objective and identify the objective function;

(c) stating all the constraints.

5. A company makes three products P_1, P_2, P_3 that go through three departments: drilling, lathe, and assembly. The time required by each product in various departments, the total available time in each department, and the profit for each product is given below:

Products	drill	lathe	assembly	Profit
P_1	3	3	8	\$100
P_2	6	5	10	\$150
P_3	7	4	12	\$200
Total (hours)	180	240	860	

Formulate but ***DO NOT SOLVE*** the problem of maximizing profit for the company. Define the variables used, state the objective function and the constraints.

6. A small petroleum company owns two refineries. Refinery 1 costs \$20,000 per day to operate, and it can produce 400 barrels of high-grade oil, 300 barrels of medium-grade oil and 200 barrels of low-grade oil each day. Refinery 2 is newer and more modern. It costs \$25,000 per day to operate, and it can produce 300 barrels of high-grade oil, 400 barrels of medium-grade oil, and 500 barrels of low-grade oil each day.
The company has orders totaling 25,000 barrels of high-grade oil, 27,000 barrels of medium-grade oil, and 30,000 barrels of low-grade oil. How many days should it run each refinery to minimize its costs and still refine enough oil to meet its orders?

(a) Define the variables.

(b) Set up the objective function and state whether it is maximized or minimized.

(c) Set up the constraints.

Solutions Word Problems

1. $x_1 = \#$ pkgs of Regular dried fruit
   $x_2 = \#$ pkgs of Special dried fruit
   $x_3 = \#$ pkgs of Deluxe dried fruit
   $P = \text{profit} = 2.25x_1 + 2.5x_2 + 2x_3$

$$\begin{cases} x_1 + 2x_2 + 2x_3 & \le 3000 \\ 2x_1 + x_2 + 2x_3 & \le 3500 \\ x_1 + x_2 + x_3 & \le 4000 \\ x_1 + x_2 + x_3 & \ge 1200 \\ x_1\ ,\ x_2\ ,\ x_3 & \ge 0 \end{cases}$$

has available, use $\le$

at least, use $\ge$

2. Transpose the table:

	Cell phone	Walkman	Modem	Totals
plastic in gm	120	90	170	200 000
production time in minutes	20	30	15	500(60)
packaging in dollars	1	1.50	1.25	1800

$x_1 = \#$ cell phone casings produced
$x_2 = \#$ walkman casings produced
$x_3 = \#$ modem casings produced

$P = \text{profit} = 6x_1 + 8x_2 + 11x_3$

$$\begin{cases} 120x_1 + 90x_2 + 170x_3 & \le 200\,000 \\ 20x_1 + 30x_2 + 15x_3 & = 30\,000 \\ x_1 + 1.5x_2 + 1.25x_3 & \le 1800 \\ x_1 + x_2 + x_3 & \ge 1200 \\ x_1\ ,\ x_2\ ,\ x_3 & \ge 0 \end{cases}$$

has available, use $\le$
operates (i.e. using ALL), use $=$

at least, use $\ge$

3. Table needs to be transposed

	I	II	III	(Products)
cashews	2	3	4	500
pecans	1	1	$\frac{1}{2}$	100
peanuts	2	1	$\frac{1}{2}$	50
Price	4.25	5	6.50	

Note: The fact that each can contains 5 pounds of nuts is irrelevant to solving!

$x_1 = \#$ cans of P-I
$x_2 = \#$ cans of P-II
$x_3 = \#$ cans of P-III

$R = \text{revenue} = 4.25x_1 + 5x_2 + 6x_3$

Standard Max Problem!

$$\begin{cases} 2x_1 + 3x_2 + 4x_3 & \le 500 \\ x_1 + 2x_2 + \frac{1}{2}x_3 & \le 100 \\ 2x_1 + x_2 + \frac{1}{2}x_3 & \le 50 \\ x_1\ ,\ x_2\ ,\ x_3 & \ge 0 \end{cases}$$

4. Transpose!

	A	B	C	Totals
Tool and Die	9	3	0.5	477
Drill Press	5	4	0	350
Lathe	3	0	2	150

$x_1 = \#$ "A" products produced per week
$x_2 = \#$ "B" products produced per week
$x_3 = \#$ "C" products produced per week

$P = \text{profit} = 30x_1 + 9x_2 + 15x_3$

Standard Max Problem

$$\begin{cases} 9x_1 + 3x_2 + 0.5x_3 & \le 477 \\ 5x_1 + 4x_2 & \le 350 \\ 3x_1 + 2x_3 & \le 150 \\ x_2 & \le 20 \\ x_1\ ,\ x_2\ ,\ x_3 & \ge 0 \end{cases}$$

has available, use $\le$

5. Transpose!

	P_1	P_2	P_3	Totals
Drill	3	6	7	180
Lathe	3	5	4	240
Assembly	8	10	12	860
Profit	100	150	200	

$x_1 = \#\ P_1$ products produced
$x_2 = \#\ P_2$ products produced
$x_3 = \#\ P_3$ products produced

$P = \text{profit} = 100x_1 + 150x_2 + 200x_3$

Standard Max Problem

$$\begin{cases} 3x_1 + 6x_2 + 7x_3 & \leq 180 \\ 3x_1 + 5x_2 + 4x_3 & \leq 240 \\ 8x_1 + 10x_2 + 12x_3 & \leq 860 \\ x_1\ ,\ x_2\ ,\ x_3 & \geq 0 \end{cases}$$

has available, use $\leq$

6. Set up the following table:

	Refinery 1	Refinery 2	Totals
High-grade oil	400	300	25000
Medium-grade oil	300	400	27000
Low-grade oil	200	500	30000
Daily costs	20000	25000	

$x_1 = \#$ days Refinery 1 should operate to meet objectives
$x_2 = \#$ days Refinery 2 should operate to meet objectives

$$\begin{cases} 400x_1 + 300x_2 & = 25000 \implies 4x_1 + 3x_2 = 250 \\ 300x_1 + 400x_2 & = 27000 \implies 3x_1 + 4x_2 = 270 \\ 200x_1 + 500x_2 & = 30000 \implies 2x_1 + 5x_2 = 300 \\ x_1\ ,\ x_2 & \geq 0 \end{cases}$$

meets (similar to "Exactly"), use $=$

$C = \text{Cost} = 20000x_1 + 25000x_2$