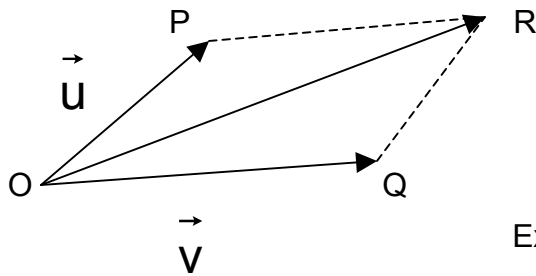


Vectors Problems

- (1) Let \vec{u} and \vec{v} be the position vectors of points P and Q respectively and let R be the terminal point of $\vec{u} + \vec{v}$



$O = \text{the origin, } \overrightarrow{OP} = \vec{u}, \overrightarrow{OQ} = \vec{v}$

Express the following in terms of \vec{u} and \vec{v} :

- (a) \overrightarrow{QP} ; (b) \overrightarrow{PQ} ; (c) \overrightarrow{RP} ; (d) \overrightarrow{QR} ; (e) \overrightarrow{RQ} ; (f) \overrightarrow{RO}

- (2) Determine whether \vec{u} and \vec{v} are parallel or not .

(a) $\vec{u} = (1, 2, -1), \vec{v} = (2, 1, 0)$

(b) $\vec{u} = (3, -6, 3), \vec{v} = (-1, 2, -1)$

(c) $\vec{u} = (1, 0, 1), \vec{v} = (-1, 0, 1)$

(d) $\vec{u} = (2, 0, -1), \vec{v} = (-8, 0, 4)$

- (3) Find a point Q such that \overrightarrow{PQ} has

(i) the same direction and the same magnitude as \vec{v} (i.e. $\overrightarrow{PQ} = \vec{v}$)

(ii) the opposite direction and the same magnitude as \vec{v} (i.e. $\overrightarrow{PQ} = -\vec{v}$)

(a) $P(-1, 2, 2), \vec{v} = (1, 2, -1)$

(b) $P(3, 0, -1), \vec{v} = (2, -1, 3)$

- (4) If $\vec{u} = (3, -1, 0), \vec{v} = (4, 0, 1), \vec{w} = (1, 1, 3)$, find \vec{x} such that :

(a) $3(2\vec{u} + \vec{x}) + \vec{w} = 2\vec{x} - \vec{v}$

(b) $2(3\vec{v} - \vec{x}) = 5\vec{w} + \vec{u} - 3\vec{x}$

- (5) Find c_1, c_2, c_3 (scalars) such that

(a) $c_1\vec{u} + c_2\vec{v} + c_3\vec{w} = (2, -1, 6)$

(b) $c_1\vec{u} + c_2\vec{v} + c_3\vec{w} = (1, 3, 0)$

where $\vec{u} = (1, 1, 2), \vec{v} = (0, 1, 2), \vec{w} = (1, 0, -1)$

- (6) Let $\vec{u} = (3, -1, 0), \vec{v} = (4, 0, 1), \vec{w} = (1, 1, 1)$, show that there does not exist c_1, c_2, c_3 (scalars) such that :

(a) $c_1\vec{u} + c_2\vec{v} + c_3\vec{w} = (1, 2, 1)$

(b) $c_1\vec{u} + c_2\vec{v} + c_3\vec{w} = (5, 6, -1)$

- (7) Let $P_1 = (2, 1, -2), P_2 = (1, -2, 0)$. Find the coordinates of P such that

(a) P is 1/5 of the way from P_1 to P_2

(b) P is 1/4 of the way from P_1 to P_2

(c) P is 1/2 of the way from P_1 to P_2 (i.e. the midpoint)

Note : P is a point on the vector $\overrightarrow{P_1P_2}$

- Answers : (1 a) $\vec{u} - \vec{v}$; (1 b) $\vec{v} - \vec{u}$; (1 c) $-\vec{v}$; (1 d) \vec{u} ; (1 e) $-\vec{u}$; (1 f) $-\vec{u} - \vec{v}$; (2 a) no ; (2 b) yes, $\vec{u} = -3\vec{v}$; (2 c) no
 (2 d) yes, $\vec{v} = -4\vec{u}$; (3 a) (i) $(0, 4, 1)$, (ii) $(-2, 0, 3)$; (3 b) (i) $(5, -1, 2)$, (ii) $(1, 1, -4)$
 (4 b) $(-16, 4, 9)$; (5 b) $c_1 = -5, c_2 = 8, c_3 = 6$; (7 a) $P(9/5, 2/5, -8/5)$;
 (7 b) $P(7/4, 1/4, -3/2)$; (7 c) $P(3/2, -1/2, -1)$