Spanning

Span $\left\{\overrightarrow{v_1},\overrightarrow{v_2},\overrightarrow{v_3},....,\overrightarrow{v_n}\right\}$ = set of all linear combinations of $\overrightarrow{v_1},\overrightarrow{v_2},\overrightarrow{v_3},....,\overrightarrow{v_n}$

Column Space of a matrix A = span of the column vectors of A

(1) Does a particular vector belong to the span of a set of vectors ?

Does $\vec{u}=\left(4,2,1\right)$ belong to the Span $\left\{\left(2,-2,-4\right),\left(-6,8,12\right),\left(-2,4,4\right)\right\}$?

Does $\vec{v}=\left(-2,-2,4\right)$ belong to the Span $\left\{\left(2,-2,-4\right),\left(-6,8,12\right),\left(-2,4,4\right)\right\}$?

This is identical to the question : Can $\vec{u}$(or $\vec{v}$) be written as a L.C. of the vectors in the set ?

(2) Find Span {group of vectors} or Col (A):

(a) Span $\left\{\left(0,0,0\right)\right\}$ (b) Span $\left\{\left(2,1,-3\right)\right\}$ (c) Span $\left\{\left(2,1,-3\right),\left(-4,-2,6\right)\right\}$

(d) Span $\left\{\left(2,1,-3\right),\left(1,0,-2\right)\right\}$ (e) Span $\left\{\left(1,0,0\right),\left(0,1,0\right),\left(0,0,1\right)\right\}$

(f) Span $\left\{\left(4,5,-8\right),\left(2,-1,4\right),\left(-1,0,7\right)\right\}$

(g) Span $\left\{\left(1,1,-2\right),\left(1,0,1\right),\left(1,5,-14\right),\left(-4,-3,5\right)\right\}$

(h) Span $\left\{\left(3,1,4\right),\left(2,-3,5\right),\left(5,-2,9\right),\left(1,4,-1\right)\right\}$

(i) Span $\left\{\left(0,0\right)\right\}$ (j) Span $\left\{\left(1,-3\right)\right\}$ (k) Span $\left\{\left(1,2\right),\left(3,4\right)\right\}$

Answers:

(1) $\vec{u}\notin$ Span {3 given vectors} , $\vec{v}\in$ Span {3 given vectors}

(2) (a) $\overrightarrow{O_v}$ (b) the line $\left(x,y,z\right)=t\left(2,1,-3\right)$ (c) see (b) (d) plane : $2x-y+z=0$

(e) $\mathbb{R}^3$ (f) $\mathbb{R}^3$ (g) plane : $x-3y-z=0$ (h) plane : $17x-7y-11z=0$

(i) $\overrightarrow{O_v}$ (j) the line $\left(x,y\right)=t\left(1,-3\right)$ (k) $\mathbb{R}^2$