Rules for Operations

Matrix Multiplications

(1) $A(BC) = A(BC)$ (association)

(2) $(A + B)C = AC + BC$ (distributive)

$A(B + C) = AB + AC$

(3) $k(AB) = (kA)B = A(kB)$ (scalars are associative and commutative)

(4) $A_{mxn} I_n = A$; $I_m A_{mxn} = A$; $IA = AI = I$ (if A is square)

(5) $A_{mxn} O_{nxp} = O_{mxp}$; $O_{qxm} A_{mxn} = O_{qxn}$

$AO = OA = O$ (if both A and O are square)

(6) AB is a diagonal matrix if both A and B are diagonal.

Transposes

(1) $(A^T)^T = A$

(2) $(A + B)^T = A^T + B^T$

(3) $(kA)^T = k(A^T)$

(4) $(AB)^T = B^T A^T$

(5) $(A^T)^3 = (A^T)(A^T)(A^T)$

$= (A^3)^T$

Inverses (assume A^{-1}, B^{-1} both exist)

(1) $(A^{-1})^{-1} = A$

(2) $(A + B)^{-1} \neq A^{-1} + B^{-1}$

(3) $(kA)^{-1} = \frac{1}{k}(A^{-1})$

(4) $(AB)^{-1} = B^{-1} A^{-1}$

(5) $(A^{-1})^3 = (A^{-1})(A^{-1})(A^{-1}) = (A^3)^{-1}$

$= A^{-3}$

(6) $(A^{-1})^T = (A^T)^{-1}$

Negative Statements (Prove these statements using 2x2 matrices with numerical entries)

(1) $AB \neq BA$

(2) $(A + B)^2 \neq A^2 + 2AB + B^2$

(3) $(A - B)^2 \neq A^2 - 2AB + B^2$

(4) $(A - B)(A + B) \neq A^2 - B^2$

(5) $(A + B)^{-1} \neq A^{-1} + B^{-1}$ (in fact $(A + B)^{-1}$ may not even exist if both A^{-1} and B^{-1} exist)

(6) $AB = O$ does not imply $A = O$ or $B = O$

(7) $A^2 = O$ does not imply $A = O$

(8) $A^2 = I$ does not imply $A = I$

(9) $AB = AC$ does not imply $B = C$

Some theoretical results - using Inverses

(1) $AB = AC$ and A^{-1} exists $\Rightarrow B = C$ (cancellation law)

(2) $AB = O$ and A^{-1} exists $\Rightarrow B = O$

(3) $A^2 = I$ and A^{-1} exists $\Rightarrow A = A^{-1}$

Square Matrices which commute (exceptions to the rule)

(1) $A A^{-1} = A^{-1} A = I$ (a matrix and its inverse)

(2) $A B = B A$ (if A and B are diagonal matrices)

(3) $A I = I A = A$ (if A is square)

(4) $A O = O A = O$ (if both A and O are square)

Square Matrices which do not have inverses

(1) O
(2) matrices with multiple rows (columns)
(3) matrices with a row of zeros
In general, matrices whose determinant is zero are not invertible.