Review Problems # 2

(Part A)

(1) Let $A=\begin{pmatrix}4&2&3\\1&-2&-4\\3&-1&-2\end{pmatrix}$, $\vec{b}=\begin{pmatrix}5\\-10\\30\end{pmatrix}$, $\vec{x}=\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$

(a) find det A , adjoint A and A^{-1} using det A and adjoint A

(b) Solve $A\vec{x}=\vec{b}$ using A^{-1}

(c) Solve for x_3 only using Cramer's Rule

(2) Let $A=\begin{pmatrix}a&b&c\\d&e&f\\g&h&i\end{pmatrix}$; det $A=-3$ and B equal a 3x3 matrix such that det $B=5$

find: (a) det $\left(A^{-1}\right)$; (b) det $\left(A^{t}\right)$; (c) det $\left(\frac{1}{3}B\right)$; (d) det $\left(2A^{3}B\right)$

(e) det $\begin{pmatrix}a&b&c\\5a+2d&5b+2e&5c+2f\\g-d&h-e&i-f\end{pmatrix}$; (f) det $\begin{pmatrix}a&b&c\\3d&3e&3f\\-2d&-2e&-2f\end{pmatrix}$

(3) Let $A=\begin{pmatrix}1&3&15k\\2&1&-5\\1&-2&7\end{pmatrix}$;

(a) find k such that det A = 0 , using (a) find k such that

(b) $A\vec{x}=\vec{O}$ has a parametric solution

(c) A^{-1} $\exists$

(d) $A\vec{x}=\vec{b}$ has a unique solution

(4) If $A^{-1}=A^{t}$, what are the possible values for det A ?

(5) Given the points A (2, -1, 3) , B (4, 0, 2) , C (-2, 2, 1)

(a) find the equation of the line through A and B

(b) determine whether the point (8, 2, 0) lies on the line in (a)

(6) Evaluate det $\begin{pmatrix}3&-6&9&-3\\0&2&-1&7\\2&-4&2&2\\3&1&2&-1\end{pmatrix}$

<u>Answers</u>:

(1 a) det A = -5 ; Adj A = $\begin{pmatrix}0&1&-2\\-10&-17&19\\5&10&-10\end{pmatrix}$; $A^{-1}=-\frac{1}{5}\begin{pmatrix}0&1&-2\\-10&-17&19\\5&10&-10\end{pmatrix}$

(1 b) $\vec{x}=A^{-1}\vec{b}=\begin{pmatrix}14\\-138\\75\end{pmatrix}$; (1 c) $x_3=\dfrac{\det A_3}{\det A}=\dfrac{-375}{-5}=75$

(2) (a) $\dfrac{1}{\det A}=-\dfrac{1}{3}$; (b) det $A=-3$; (c) $\left(\dfrac{1}{3}\right)^{3}$ det $B=\dfrac{5}{27}$; (d) $2^{3}\left(\det A\right)^{3}$ det $B=-1080$

(e) -6 ; (f) 0 (multiple rows)

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Answers:

(3) (a) det $A = 0 \Rightarrow k = -\frac{4}{5}$; (b) $A\vec{x} = \vec{O}$ has a parametric solution $\Leftrightarrow$ det $A = 0 \to k = -\frac{4}{5}$;
(c) $A^{-1}\exists \Leftrightarrow$ det $A \neq 0 \to k \neq -\frac{4}{5}$; (d) $A\vec{x} = \vec{b}$ has a unique solution $\Leftrightarrow$ det $A \neq 0 \to k = -\frac{4}{5}$

(4) det $A = \pm 1$

(5) (a) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$; (b) yes! let t = 3 (6) 624

or use B or any multiples

(Part B)

(1) Let $A = \begin{pmatrix} 3 & 0 & -6 \\ -4 & -1 & 1 \\ 1 & 1 & 2 \end{pmatrix}; \vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}; \vec{b} = \begin{pmatrix} 6 \\ -3 \\ 12 \end{pmatrix}$

(a) find det A , adj A and A^{-1} using det A and adj A

(b) Solve $A\vec{x} = \vec{b}$ using A^{-1}

(2) Evaluate det $\begin{pmatrix} 3 & 1 & -2 & 4 \\ 1 & -2 & -3 & 2 \\ -2 & 4 & 8 & 3 \\ 0 & 2 & -2 & 4 \end{pmatrix}$

(3) Let $A = \begin{pmatrix} a & b & c \\ r & s & t \\ x & y & z \end{pmatrix}$ and det $A = 2$;

(i) find: (a) det (AA^t) ; (b) det $(-A^2)$; (c) det $(3A)^{-1}$; (d) det $\begin{pmatrix} a & b & c \\ 3x & 3y & 3z \\ r+2x & s+2y & t+2z \end{pmatrix}$; (e) C_{32}

(ii) How many solutions can $A\vec{x} = \vec{O}$ have ? Why ?
(iii) What is the rank of A ?

(4) Given the points A (1, 2, -4) , B (2, 1, -3) , C (0, 1, 2)
(a) find an equation of the line through A and B
(b) find an equation (in standard form) of the plane containing A , B and C

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(5) An economy consists of two industries, coal and steel. Production of \$1 of worth of coal requires an input of \$0.40 of coal and \$0.20 of steel. Production of \$1 of worth of steel requires an input of \$0.60 of coal and \$0.50 of steel. Find the total production needed to satisfy a demand of \$20 billion of coal and \$10 billion of steel.

(a) Define your variables in words.

(b) Find $C, I - C$ and solve the problem.

(c) Is the economy productive? Why or why not?

(d) Which, if any, of the two industries is profitable? Explain.

(e) Find the internal consumption.

Answers:

(1) (a) det $A = 9$, adj $A = \begin{pmatrix} -3 & -6 & -6 \\ 9 & 12 & 21 \\ -3 & -3 & -3 \end{pmatrix}$; $A^{-1} = \frac{1}{9}\begin{pmatrix} -3 & -6 & -6 \\ 9 & 12 & 21 \\ -3 & -3 & -3 \end{pmatrix}$; (b) $\vec{x} = A^{-1}\vec{b} = \begin{pmatrix} -8 \\ 30 \\ -5 \end{pmatrix}$; (2) -260

(3) (i) (a) $(\text{det } A)(\text{det } A^t) = (\text{det } A)^2 = 4$; (b) $(-1)^3 (\text{det } A)^2 = -4$; (c) $\frac{1}{\text{det }(3A)} = \frac{1}{3^3 \text{ det } A} = \frac{1}{(27)(2)} = \frac{1}{54}$

(d) -6 ; (e) $-\text{M}_{32} = -\begin{vmatrix} a & c \\ r & t \end{vmatrix} = -(at - rc)$; (ii) $\vec{x} = \vec{O}$ only since det $A \neq 0$; (iii) rank $A = 3$ since $A \sim \text{I}_3$ (det $A \neq 0$)

(4) (a) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$; (b) $5x + 7y + 2z - 11 = 0$

(5) (a) $x =$ the total \$ value of coal produced to meet both internal and external requirements.
$y =$ the total \$ value of steel produced to meet both internal and external requirements.

(b) $C = \begin{bmatrix} .4 & .6 \\ .2 & .5 \end{bmatrix}$; $I - C = \begin{bmatrix} .6 & -.6 \\ -.2 & .5 \end{bmatrix}$ then $X = (I - C)^{-1} D = \begin{bmatrix} 88.89 \\ 55.56 \end{bmatrix}$ in Billions

(c) yes! $X = \underbrace{(I - C)^{-1}}_{\text{positive entries}} \underbrace{D}_{\text{positive entries}} \Rightarrow X \geq 0$

(d) Coal → costs .60 to produce \$1 worth of coal → profit = .40 per \$1

(e) $CX = \underbrace{\begin{pmatrix} 88.89 \\ 55.56 \end{pmatrix}}_{\text{Total}} - \underbrace{\begin{pmatrix} 20 \\ 10 \end{pmatrix}}_{\text{External}} = \underbrace{\begin{pmatrix} 68.89 \\ 45.56 \end{pmatrix}}_{\text{internal}}$