

## Linear Combinations

Given a set of vectors  $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \dots, \vec{v}_n\}$ ;  $\vec{u}$  is a linear combination of the  $v$ 's if there exists  $c_1, c_2, c_3, \dots, c_n$  such that  $c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 + \dots + c_n\vec{v}_n = \vec{u}$

$\text{Span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \dots, \vec{v}_n\}$  = the set of all vectors which are Linear Combinations of the  $v$ 's .

If  $\vec{u}$  is a L.C. of the  $v$ 's then  $\vec{u} \in \text{Span}\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \dots, \vec{v}_n\}$

(1) Given  $\vec{u} = (1, 2, 3)$  and  $\vec{v} = (4, 5, 6)$  or  $\{(1, 2, 3), (4, 5, 6)\}$ , determine whether the following vectors are L.C.'s of  $\vec{u}$  and  $\vec{v}$  .

(a)  $(0, 0, 0)$ ; (b)  $(1, 2, 3)$ ; (c)  $(8, 10, 12)$ ; (d)  $(7, 8, 9)$ ; (e)  $(5, 7, 9)$   
(f)  $(2, -1, 5)$

(2) Given  $\vec{u} = (4, 0, -2, -2)$ ,  $\vec{v} = (1, -1, 2, 3)$ ,  $\vec{w} = (0, 2, 1, 4)$

Determine whether each of the following vectors belongs to the  $\text{Span}\{\vec{u}, \vec{v}, \vec{w}\}$  .

(a)  $(6, -8, -1, -8)$ ; (b)  $(0, 0, 0, 0)$ ; (c)  $(6, 0, 3, 8)$ ; (d)  $(-1, 5, 7, 1)$

## Linear Independence vs Linear Dependence

A set of vectors  $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \dots, \vec{v}_n\}$  is a L.I. if  $c_1\vec{v}_1 + c_2\vec{v}_2 + c_3\vec{v}_3 + \dots + c_n\vec{v}_n = \vec{0}$

$\Rightarrow c_1 = c_2 = c_3 \dots c_n = 0$  (uniquely) . Otherwise,  $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \dots, \vec{v}_n\}$  is L.D.

Determine whether the following sets of vectors are LI or LD.

(1)  $\{\vec{u}, \vec{v}, \vec{w}\}$  where  $4\vec{u} - 2\vec{v} + 3\vec{w} = \vec{0}$       (2)  $\{\vec{u}, \vec{v}, \vec{w}\}$  where  $0\vec{u} - 0\vec{v} - 3\vec{w} = \vec{0}$

(3)  $\{\vec{u}, \vec{v}, \vec{w}\}$  where  $0\vec{u} + 0\vec{v} + 0\vec{w} = \vec{0}$  and this is the only way of writing this equation

(4)  $\{\vec{u}, \vec{v}, \vec{w}, \vec{0}\}$       (5)  $\{(1, 2, 3), (2, 4, 6)\}$       (6)  $\{(1, 2, 3), (4, 5, 6), (7, 8, 9)\}$

(7)  $\{(1, 2, 3), (0, 2, 5), (1, 0, 4)\}$       (8)  $\{(-2, 0, 1), (3, 2, 5), (6, -1, 1), (7, 0, -2)\}$

(9)  $\{(1, 1, 0, -1), (0, 0, 2, 2), (3, 3, 0, 0)\}$

(10)  $\{(3, 0, -3, 6), (0, 2, 3, 1), (0, -2, -2, 0), (-2, 1, 2, 1)\}$

Answers: LI : (3) , (7) , (9) , (10) ;      LD : (1) , (2) , (4) , (5) , (6) , (8)