Determinants

(1) $\det [a] = a$; (2) $\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} =$; (3) $\det \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$

(4) (a) $\det 0 =$; (b) $\det I =$; (c) $\det D =$ (where D is a diagonal matrix)
(d) det (△ matrix) = ; (e) det (E) = (where E is elementary matrix)

(5) If B is a matrix obtained by performing a row operation (interchange) on matrix A , then
det B = ____ det A and det A = ____ det B

(6) If B is a matrix formed by performing a row operation of form $kR_i + R_j \to R_j$ (comparable to using " 1 " as a pivot) on matrix A , then det B = ____ det A .

(7) If B is a matrix formed by performing a row operation of form $kR_i + mR_j \to R_j$ (" m " as a pivot) on matrix A , then det B = ____ det A and det A = ____ det B .

(8) If a row of A has a common factor " k " ,
how does the " k " affect the evaluation of det A ?

(9) If <u>every</u> row of A has a common factor " k " ,
how does the " k " affect the evaluation of det A ?

(10) If A has a row of zeros , then det A =

(11) $\det (A^t) =$

(12) Column operations can be used in evaluating det A as a consequence of # 11 .

(13) det (AB) =

(14) $\det A^{-1} =$

(15) If A has multiple rows (columns) , then det A =

(16) Cofactor expansion : cofactor $= c_{ij}$; Minor $= M_{ij}$

Determinants Problems

(1) Compute :

(a) $\det\begin{pmatrix} 5 & -1 & 2 & -1 \\ 3 & 1 & -1 & 2 \\ 0 & 1 & 2 & 2 \\ 1 & 2 & -1 & 1 \end{pmatrix}$ (b) $\det\begin{pmatrix} 2 & 4 & 6 & 8 \\ -5 & 6 & -7 & 8 \\ -4 & -3 & 2 & 1 \\ 6 & 5 & 1 & 7 \end{pmatrix}$

(c) $\det\begin{pmatrix} 4 & -1 & 3 & -1 \\ 3 & 1 & 0 & 2 \\ 0 & 1 & 2 & 2 \\ 1 & 2 & -1 & 1 \end{pmatrix}$ (d) $\det\begin{pmatrix} 2 & 4 & 6 & 8 \\ 5 & 6 & 7 & 8 \\ -4 & -3 & 2 & 1 \\ 6 & 5 & 1 & -7 \end{pmatrix}$

Answers : (a) -69 ; (b) 1816 ; (c) -56 ; (d) 272

(2) Given det A = -2 and det B = 3 , A and B are 5x5 matrices ;

Compute : (a) $\det(A^2 B)$; (b) $\det(A^3)$; (c) $\det A^{-1}$; (d) $\det(A^{-1} B A)$; (e) $\det\left(-\frac{1}{3}A\right)$

Answers : (a) 12 ; (b) –8 ; (c) –1/2 ; (d) 3 ; (e) 2/243

(3) Given det A = 3 and det B = 2 , A and B are 4x4 matrices ;

Compute : (a) $\det(A B^2)$; (b) $\det(A^3)$; (c) $\det A^{-1}$; (d) $\det(A^{-1} B A)$; (e) $\det\left(\frac{1}{2}A\right)$

Answers : (a) 12 ; (b) 27 ; (c) 1/3 ; (d) 2 ; (e) 3/16

(4) Assume $\det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = 3$. Compute the determinant of each following matrices :

(a) $B = \begin{pmatrix} a+7c & 2b & c \\ d+7f & 2e & f \\ g+7i & 2h & i \end{pmatrix}$ (b) $C = \begin{pmatrix} a & d & 5g \\ b & e & 5h \\ c & f & 5i \end{pmatrix}$ (c) $D = \begin{pmatrix} g & h & i \\ a & b & c \\ d & e & f \end{pmatrix}$

(d) $E = \begin{pmatrix} a & b & c \\ d & e & f \\ 3a & 3b & 3c \end{pmatrix}$ (e) $F = \begin{pmatrix} a & b & c \\ 4a+5d & 4b+5e & 4c+5f \\ \frac{1}{2}g & \frac{1}{2}h & \frac{1}{2}i \end{pmatrix}$

Answers : (a) 6 ; (b) 15 ; (c) 3 ; (d) 0 ; (e) 15/2

Determinants Problems

(5) Assume $\det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = -4$, compute the determinant of the following matrices:

(a) $A = \begin{pmatrix} 2a & b+9c & c \\ 2d & e+9f & f \\ 2g & h+9i & i \end{pmatrix}$ (b) $B = \begin{pmatrix} a & 5d & g \\ b & 5e & h \\ c & 5f & i \end{pmatrix}$ (c) $C = \begin{pmatrix} g & h & i \\ a & b & c \\ d & e & f \end{pmatrix}$

(d) $D = \begin{pmatrix} 2a & 2b & 2c \\ d & e & f \\ -3a & -3b & -3c \end{pmatrix}$ (e) $E = \begin{pmatrix} a & b & c \\ -4a+5d & -4b+5e & -4c+5f \\ \frac{1}{2}g & \frac{1}{2}h & \frac{1}{2}i \end{pmatrix}$

Answers: (a) -8 ; (b) -20 ; (c) -4 ; (d) 0 ; (e) -10

(6) Assume A , B are square matrices of the SAME size.
(a) If A^{-1} exists; prove $\det(A^{-1}BA) = \det B$
(b) Prove $\det(AB) = \det(BA)$
Use Lhs = ______ ; Rhs = _______

(7) Evaluate determinant A by inspection. Give a reason for your response.

(a) $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ (b) $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ (c) $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

(d) $A = \begin{pmatrix} -2 & 8 & 1 & 4 \\ 3 & 2 & 5 & 1 \\ 1 & 10 & 6 & 5 \\ 4 & -6 & 4 & -3 \end{pmatrix}$ (e) $A = \begin{pmatrix} -3 & 0 & 1 \\ 5 & 0 & 6 \\ 8 & 0 & 3 \end{pmatrix}$

(8) Show that $\det\begin{pmatrix} b+c & c+a & b+a \\ a & b & c \\ 1 & 1 & 1 \end{pmatrix} = 0$ using Row Operations