

BASIS

1. Find a basis for each of the following subspaces and state the dimension of each.

(a) $S_1 = \text{Span}\{(1, 1, -3), (2, 2, -6), (-4, -4, 12)\}$; (b) $S_2 = \{(x, y, z) \in \mathbb{R}^3 \mid y = 0\}$

(c) $S_3 = \{(x, y, z) \in \mathbb{R}^2 \mid x + 2y = 0\}$; (d) $S_4 = \text{Span}\{(1, 1, 2), (-1, 2, 5), (0, 3, 7), (1, 4, 9)\}$

(e) $S_5 = \{(x, y, z) \in \mathbb{R}^3 \mid 2x + y - 3z = 0\}$; (f) $S_6 = \text{Span}\{(1, 1, -3), (2, 2, -6), (5, 6, 9)\}$

2. None of the following is a basis for \mathbb{R}^3 . Why not?

(a) $\{(5, 7, -2), (-4, 6, 9)\}$; (b) $\{(0, 0, 0), (1, 2, 3), (4, 5, 6)\}$

(c) $\{(3, 5, -7), (-2, 4, 1), (5, 1, -8)\}$; (d) $\{(1, 1, 0), (0, 1, 1), (1, 0, 1), (1, 1, 1)\}$

3. (a) What is the standard basis for \mathbb{R}^4 ?; (b) Find another basis for \mathbb{R}^4 .

4. Find a basis for the following subspaces. State the dimension of S .

(a) $S = \{(x, 0) \in \mathbb{R}^2\}$; (b) $S = \{(x, y) \in \mathbb{R}^2 \mid x + 2y = 0\}$; (c) $S = \{(x, y, z) \in \mathbb{R}^3 \mid z = 0\}$

(d) $S = \{(-t, t, 0) \mid t \in \mathbb{R}\}$; (e) $S = \{(x, y, z) \in \mathbb{R}^3 \mid x - y - z = 0\}$

(f) $S = \{(x, y, z) \in \mathbb{R}^3 \mid 4x + y - z = 0\}$; (g) $S = \{(t, t, t) \mid t \in \mathbb{R}\}$

(h) $S = \{(x, y, z, w) \in \mathbb{R}^4 \mid x + 3y = 0, z + w = 0\}$

(i) $S = \{(x, y, z, w) \in \mathbb{R}^4 \mid x + 2y - z + 5w = 0\}$

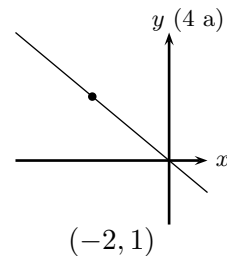
(j) $S = \{(x, y, z, w) \in \mathbb{R}^4 \mid x + y = z + w\}$; (k) $S = \{(a + b, a - b, a, b) \in \mathbb{R}^4\}$

(l) $S = \{(a, a + b, a - b, b) \in \mathbb{R}^4\}$; (m) $S = \{(a - b, b + c, a, b + c) \in \mathbb{R}^4\}$

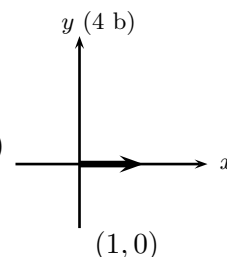
(n) $S = \{(0, 0, 0) \in \mathbb{R}^3\}$; (o) \mathbb{R}^3 ; (p) \mathbb{R}^4

BASIS

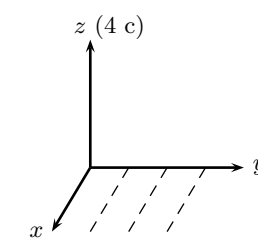
(4 a) basis: $\{(1, 0)\}$; $d = 1 \rightarrow$ this is a line in \mathbb{R}^2 (the x -axis)



(4 b) basis: $\{(-2, 1)\}$; $d = 1 \rightarrow$ a line in \mathbb{R}^2

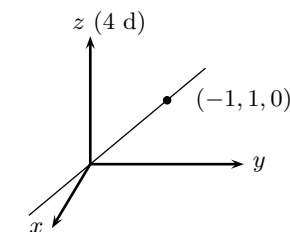


(4 c) basis: $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$; $d = 2 \rightarrow$ a plane in $\mathbb{R}^2 \rightarrow xy$ plane (floor)

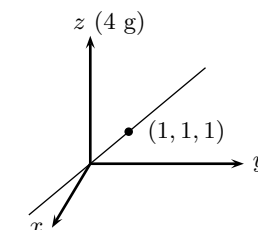


(4 d) basis: $\left\{ \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \right\}$; $d = 1 \rightarrow$ this is a line in \mathbb{R}^3

(4 e) basis: $\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\}$; $d = 2 \rightarrow$ a plane in \mathbb{R}^3



(4 f) basis: $\left\{ \begin{pmatrix} -1/4 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1/4 \\ 0 \\ 1 \end{pmatrix} \right\}$; $d = 2 \rightarrow$ a plane in \mathbb{R}^3



(4 g) basis: $\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$; $d = 1 \rightarrow$ line in \mathbb{R}^3

(4 h) basis: $\left\{ \begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -1 \\ 1 \end{pmatrix} \right\}$; $d = 2$; $\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} -3t \\ t \\ -s \\ s \end{pmatrix}$; a 2-dimensional subspace of \mathbb{R}^4

(4 i) basis: $\left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -5 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$; $d = 3$; $\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} -2r + s - 5t \\ r \\ s \\ t \end{pmatrix} = r \begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} -5 \\ 0 \\ 0 \\ 1 \end{pmatrix}$;

a 3-dimensional subspace of \mathbb{R}^4