

201-203-RE - Practice Set #20: Applications of Geometric Series

- (1) A deposit of 35 dollars is made at the beginning of each month, for a period of 4 years, in an account that pays 1.5% interest, compounded monthly. Find the balance in the account at the end of the 4 years.
- (2) The annual net profit for a company, from 1995 to 2005, can be approximated by the model $a_n = 10e^{0.2n}$, $n = 1, 2, 3, \dots, N$, where a_n is the annual net profit in millions of dollars, and n represents the year, with $n = 1$ corresponding to 1995. Estimate the total net profit during this period.
- (3) A deposit of 15 dollars is made at the beginning of each month, for a period of 5 years, in an account that pays 0.9% interest, compounded monthly. Find the balance in the account at the end of the 5 years.
- (4) A deposit of 20 dollars is made every 3 months, for a period of 10 years, in an account that pays 1% interest, compounded quarterly. Find the balance in the account at the end of the 10 years.
- (5) A deposit of \$500 can be made with two options: earn 2% interest, compounded quarterly for 3 years, or earn 1.5%, compounded monthly for 3 years. Find the balance after 3 years of both options.
- (6) A deposit of \$1200 can be made with two options: earn 1.6% interest, compounded every 2 months for 5 years, or earn 2.3% interest, compounded semi-annually for 5 years. Find the balance after 5 years of both options.
- (7) The annual net profit for a company, from 1995 to 2000, can be approximated by the model $a_n = 5e^{0.1n}$, with $n = 1, 2, 3, \dots, N$, where a_n is the annual net profit (in millions of dollars), and n represents the year, with $n = 1$ corresponding to 1995. Estimate the total net profit earned during this period.
- (8) To create a scholarship of \$600 that is to be awarded every year, we can use the series $\sum_{n=1}^{+\infty} 600e^{-0.05n}$ to determine the sum of money that has to be deposited in an account that earns 5% interest, compounded continuously. Find the amount of money that must be deposited.
- (9) To create a scholarship of \$1200 that is to be awarded every year, we can use the series $\sum_{n=1}^{+\infty} 1200e^{-0.03n}$ to determine the sum of money that has to be deposited in an account that earns 3% interest, compounded continuously. Find the amount of money that must be deposited.
- (10) A patient is given 50mg of a drug daily, for a long period of time. The amount of drug left in the patient's body after n years is given by $\sum_{k=1}^n 50e^{-k/2}$. Find the amount of drug in the patient's system after 4 years and after 15 years.
- (11) A deposit of 10 dollars is made at the beginning of each month, for a period of 6 years, into an account that pays 1% yearly interest, compounded 12 times per year. Find the balance in the account at the end of the 6 years.
- (12) The annual net profit for a company, from 1997 to 2003, can be approximated by the model $a_n = 15e^{0.3n}$, with $n = 1, 2, 3, \dots, N$, where a_n is the annual net profit (in millions of dollars), and n represents the year, with $n = 1$ corresponding to 1997. Estimate the total net profit earned during this period.
- (13) A deposit of 30 dollars is made at the beginning of each month, for a period of 7 years, into an account that pays 1.1% yearly interest, compounded 12 times per year. Find the balance in the account at the end of the 7 years.

- (14) The annual net profit for a company, from 1996 to 2001, can be approximated by the model $a_n = 6e^{0.05n}$, with $n = 1, 2, 3, \dots, N$, where a_n is the annual net profit (in millions of dollars), and n represents the year, with $n = 1$ corresponding to 1996. Estimate the total net profit earned during this period.
- (15) A deposit of 50 dollars is made bi-weekly, for a period of 8 years, into an account that pays 1.3% yearly interest, compounded every two weeks. Find the balance in the account at the end of the 8 years.
- (16) A deposit of \$700 can be made with two options: account A earns 1.8% yearly interest, compounded every two weeks, and account B earns 2.1% yearly interest, compounded monthly. The amount will stay in the account for a period of 4 years. What would the balance be after the 4 years, for both accounts?
- (17) A deposit of \$1500 can be made with two options: account A earns 1.4% yearly interest, compounded quarterly, and account B earns 1.9% yearly interest, compounded monthly. The amount will stay in the account for a period of 6 years. What would the balance be after the 6 years, for both accounts?
- (18) To create a scholarship of \$750 that is to be awarded every year, we can use the series $\sum_{n=1}^{+\infty} 750e^{-0.04n}$ to determine the sum of money that has to be deposited in an account that earns 4% interest, compounded continuously. Find the amount of money that must be deposited.
- (19) To create a scholarship of \$1500 that is to be awarded every year, we can use the series $\sum_{n=1}^{+\infty} 1500e^{-0.035n}$ to determine the sum of money that has to be deposited in an account that earns 3.5% interest, compounded continuously. Find the amount of money that must be deposited.
- (20) A patient is given 40mg of a drug daily, for a long period of time. The amount of drug left in the patient's body after n years is given by $\sum_{k=1}^n 40e^{-k/5}$. Find the amount of drug in the patient's system after 6 years and after 20 years.

In problems 21 - 34, geometric series to write the following repeating decimals as fractions.

- (21) $4.\overline{13}$ (23) $3.\overline{09}$ (25) $5.\overline{011}$ (27) $7.\overline{34}$ (29) $1.\overline{06}$ (31) $8.\overline{01}$ (33) $5.\overline{25}$
- (22) $6.\overline{04}$ (24) $2.\overline{02}$ (26) $10.\overline{23}$ (28) $20.\overline{021}$ (30) $2.\overline{03}$ (32) $2.\overline{22}$ (34) $12.30\overline{2}$
- (35) Valery would like to save \$1 400 000 for her retirement. Her account has an annual interest rate of 2.7%, compounded semi-annually. If Valery wants to retire in 40 years, how much should she be saving semi-annually?
- (36) When she retires, Liu Zhang purchases an annuity which pays her \$ 3 000 per month for 40 years. If the annuity earns 3.1% interest compounded monthly, what was the initial value of the investment?
- (37) Sobolev would like to buy a \$20 000 car, but he cannot afford to do so outright. He goes on the automaker's website and sees that he can expect to finance it at 3.5% interest compounded twice a month. If he chooses to finance the car for a six year term, what will his payments be, if he is paying twice per month?
- (38) Suzana would like to save a \$40 000 down payment for a house. If she can invest at 1.6% interest compounded weekly over ten years, how much should she save each week?

- (39) Lewis-Charles is at the car dealership and is being told his new car will cost him \$1000 every two months for four years. If he financed at 0.9% interest compounded every two months, how much is the car (not including interest)?
- (40) Chef Tony bought a commercial building that is worth \$3 000 000. He finances it on a 25 year term at 3.2% annual interest compounded semi-annually. What is his mortgage payment every 6 months?
- (41) Jennifer wants to save \$15 000 for a new car in eight years. If she can save at 1.1% interest compounded monthly, what should be her monthly deposit?
- (42) Ichiban has \$1 800 000 saved when he retires. He uses all that money to purchase an annuity earning 2.6% interest compounded every two weeks that will pay him a lump sum every two weeks for 30 years. How much will he receive every two weeks?
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ANSWERS:

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|----------------------------|-----------------------------|-------------------|
| (1) \$1732.47 | (15) \$10 962.64 | (29) 35/33 |
| (2) \$442.712 millions | (16) \$752.24 or \$761.28 | (30) 67/33 |
| (3) \$920.89 | (17) \$1631.20 or \$1680.98 | (31) 793/99 |
| (4) \$842.36 | (18) \$18 377.50 | (32) 20/9 |
| (5) \$530.84 or \$523.00 | (19) \$42 111.52 | (33) 520/99 |
| (6) \$1299.81 or \$1345.36 | (20) 126.25mg and 177.36mg | (34) 2768/225 |
| (7) \$43.195 millions | (21) 409/99 | (35) \$9695.05 |
| (8) \$11 702.50 | (22) 598/99 | (36) \$824 693.47 |
| (9) \$39 403.00 | (23) 34/11 | (37) \$154.08 |
| (10) 66.64mg and 77.03mg | (24) 200/99 | (38) \$70.92 |
| (11) \$742.34 | (25) 5006/999 | (39) \$23 555.79 |
| (12) \$414.738 millions | (26) 307/30 | (40) \$87 620.56 |
| (13) \$2620.71 | (27) 661/90 | (41) \$149.41 |
| (14) \$43.041 millions | (28) 6607/330 | (42) \$3 324.62 |