201-203-RE - Practice Set #2: Initial Value Problems

(1) Given $f''(x) = 30x^4 + 12x$; $f'(0) = 5$; $f(0) = -7$, find $f(x)$.

(2) Given $f''(x) = 24x^2 - 18x$; $f'(-1) = 2$; $f(1) = 4$, find $f(x)$.

(3) Given $f''(x) = 60\sqrt{x} - 48x$; $f'(1) = 25$; $f(4) = 30$, find $f(x)$.

(4) Find the cost function given $\dfrac{dC}{dx} = 5x - \dfrac{1}{x}$ and 10 units cost \$94.20.

(5) Find the cost function given $\dfrac{dC}{dx} = \dfrac{1}{x} + 2x$ and 7 units cost \$58.40.

(6) Find the demand function at $x = 90$ given $\dfrac{dR}{dx} = x^2 - 2x + 3$.

(7) Find the profit function at $x = 100$ given that $\dfrac{dP}{dx} = 2x + 20$ and profit on 20 items is $-\$50$.

(8) Suppose $f''(x) = 6x + 2$, and that at the point (-1,3) the slope of the tangent line is 2. Find $f(x)$.

(9) Find the equation of the curve that passes through (1,3) if its slope is given by $\dfrac{dy}{dx} = 12x^2 - 12x$ for each x.

(10) Given $\dfrac{dy}{dt} = \dfrac{\sqrt{t^3} - t}{\sqrt{t^3}}$, find the function y that satisfies the condition $y(9) = 4$.

(11) Given $\dfrac{dy}{dx} = 2x^{-2} + 3x^{-1} - 1$, find the function y that satisfies the condition $y(1) = 0$.

(12) Given $f''(x) = 18x - 6x^2$, find the function $f(x)$ that satisfies the conditions $f'(1) = 20$ and $f(1) = 15$.

(13) Suppose $f''(x) = 14 - 12x$, and that at the point (2,3) the slope of the tangent line to the graph is 5. Find $f(x)$.

(14) Find the equation of the curve that passes through (-1,20) if its slope is given by $\dfrac{dy}{dx} = 48x - 6x^2$ for each x.

(15) Given $\dfrac{dy}{dt} = \dfrac{\sqrt[3]{t^2} - 4}{\sqrt[3]{t^2}}$, find the function y that satisfies the condition $y(-8) = 4$.

(16) Given $\dfrac{dy}{dx} = 4x^{-3} + 5x^{-1} + 3$, find the function y that satisfies the condition $y(1) = 3$.

(17) Given $f''(x) = 12x^2 - 6x$, find the function $f(x)$ that satisfies the conditions $f'(1) = 8$ and $f(-1) = 5$.

(18) Find the average cost function given that the marginal cost is $0.3x^2 + 6x + 100$ and that 10 units cost \$3000.

(19) Find the demand function given that the marginal revenue is $9x^2 + 0.1x + 500$ and that the revenue from 10 units is \$8500.

(20) Find the demand function given that the marginal revenue is $9x^2 + 0.1x + 500$ and that the revenue from 10 units is \$8500.

(21) Find the demand function at $x = 16$ units given that the marginal revenue is $6\sqrt{x} + 8x + 500$.

(22) Given $f''(x) = 20x^3 - 18x + 4$; $f(1) = 4$; $f(-1) = 14$, find $f(x)$.

(23) Given $f''(x) = 2x + 10$; $f(1) = -1$; $f(2) = 15$, find $f(x)$.

(24) Given $f''(x) = 6x^2 - 6x + 1$; $f(1) = 3$; $f(-2) = 36$, find $f(x)$.

(25) Given $f''(x) = 40x^3 - 12x^2 + 18x - 10$; $f(0) = -2$; $f(1) = 5$, find $f(x)$.

ANSWERS:

(1) $f(x) = x^6 + 2x^3 + 5x - 7$

(2) $f(x) = 2x^4 - 3x^3 + 19x - 14$

(3) $f(x) = 16x^{5/2} - 8x^3 + 9x - 6$

(4) $C = \frac{5}{2}x^2 - \ln|x| - 153.50$

(5) $C = \ln|x| + x^2 + 7.45$

(6) $p(90) = \$2613$

(7) $P(100) = \$11150$

(8) $f(x) = x^3 + x^2 + x + 4$

(9) $y = 4x^3 - 6x^2 + 5$

(10) $y = t - 2\sqrt{t} + 1$

(11) $y = 3\ln|x| - x - \frac{2}{x} + 3$

(12) $f(x) = -\frac{1}{2}x^4 + 3x^3 + 13x - \frac{1}{2}$

(13) $f(x) = -2x^3 + 7x^2 + x - 11$

(14) $y = 24x^2 - 2x^3 - 6$

(15) $y = t - 12\sqrt[3]{t} - 12$

(16) $y = 5\ln|x| + 3x - \frac{2}{x^2} + 2$

(17) $f(x) = x^4 - x^3 + 7x + 10$

(18) $\overline{C} = 0.1x^2 + 3x + 100 + \frac{1600}{x}$

(19) $p = 3x^2 + 0.05x + 500 + \frac{495}{x}$

(20) $C = 4x^3 + 10e^{2x} + 990$

(21) $p(16) = \$580$

(22) $f(x) = x^5 - 3x^3 + 2x^2 - 3x + 7$

(23) $f(x) = \frac{1}{3}x^3 + 5x^2 - \frac{4}{3}x - 5$

(24) $f(x) = \frac{1}{2}x^4 - x^3 + \frac{1}{2}x^2 - 5x + 8$

(25) $f(x) = 2x^5 - x^4 + 3x^3 - 5x^2 + 8x - 2$