201-203-RE - Practice Set #15: Applications of Differential Equations

(1) The rate of increase of a population P of a town is proportional to the time t (the number of years after the year 1976) and inversely proportional to the population size P. In 1976 the population was 10 000 and in 1986 it was 20 000. In what year will the population be 52 000?

(2) The rate of increase of a population P of a city is proportional to the product of the population size P and time t (number of years after 1950). In 1950 the population was 1 500 000. The population grew by 5% after 10 years. What was the population of the city in the year 1970?

(3) A company has their sales volume S (in millions of dollars) increasing at a rate inversely proportional to the square root of time t (in years). At the present, the sales volume is 40 millions of dollars. The company predicts that in one year, the sales will be 50 millions of dollars. Find the sales volume 4 years later.

(4) A metal company has their production increasing at a rate proportional to the product of the number of units N and the time t in years. 400 units are presently produced. In 2 years, 1200 units are expected. What will be the production in 4 years?

(5) A minivan is bought for \$32 000. The rate of depreciation of the value V (in dollars) at time t (in years) is proportional to the present value V. After 3 years, the vehicle is worth \$25 600. What is the value of the minivan 6 years later?

(6) A piece of machinery is worth \$1600. The rate of depreciation of the value V (in dollars) at time t (in years) is proportional to the square root of its value V. The piece of machinery will be worth \$900 two years later. When will the piece of machinery be worth \$625?

(7) A rumor starts, with 20 people (in a population of 8500) having heard it. The rate at which the rumor spreads at time t in weeks is proportional to the number of people N who have not heard the rumor. At the end of 2 weeks, 1000 people heard the rumor. How many weeks will it take for 4000 people to hear the rumor?

(8) The production of N units is increasing at a rate proportional to the product of the number of units N and the time t (in years). Initially 100 units are produced, and 250 units after 1 year. How long will it take to produce 500 units?

(9) The rate of decomposition at time t (in hours) of a substance N is proportional to the amount of substance present. If 70% of the initial amount of substance has decomposed after 4 hours, find the remaining amount of substance after 8 hours, if initially the amount of substance is 120g.

(10) The rate of increase of a population P of a city, where t is the number of years after 1965, is inversely proportional to the square root of the population size. In 1965, the population was 8 100, and in 1985 it was 22 500. In how many years will the population reach 44 100 people?

(11) The rate of decay at time t (in hours) of a radioactive substance N is proportional to the amount of substance present. If 80% of the initial amount of radioactive substance remains after 2 hours, find the remaining amount of radioactive substance at time t if initially the amount was 10g.

(12) The rate of decay at time t (in hours) of a radioactive substance N is proportional to the amount of substance present. If 65% of the initial amount of radioactive substance has decomposed after 5 hours, find the remaining amount of radioactive substance after 8 hours if initially the amount was 100g.

(13) A software company has their sales volume S (in millions of dollars) increasing at a rate proportional to the square root of both the sales volume S and the time t (in years). The company started with a sales volume of 36 millions of dollars. A year later, the sales volume reached 64 millions of dollars. When will the sales volume reach 484 millions of dollars?

(14) A company has their production increasing at a rate proportional to the product of the number of units N and the time t in years. 16 units are presently produced. In 1 year, 64 units are expected. In how many years will the production be 1024 units?

(15) A piece of furniture is worth \$2500. The rate of depreciation of the value V (in dollars) at the time t (in years) is proportional to the square of the value. The piece of furniture is worth \$1600 two years later. How much will the piece of furniture be worth after 3 years?

(16) The rate of increase of a population size P in a city is proportional to the population size, and inversely proportional to the time t (in years). After 1 year, the population is 75 000, and after 5 years it is 225 000. What is the population size after 20 years?

(17) A rumor starts spreading, with 100 persons who know it, in a population of 100 000. The rumor spreads at a rate that is proportional to the time t (in weeks), and inversely proportional to the number of persons N who have heard the rumor. At the end of 5 weeks, 10 000 persons have heard the rumor. How many persons will have heard the rumor after 25 weeks?

(18) The number of units produced N is increasing at a rate that is proportional to the time t (in years), and inversely proportional to the number of units. Initially, 1000 units are produced, and that number increased to 2000 units after one year. What will be the production after 4 years?

(19) A company has their production level increasing at a rate that is proportional to the product of the number of units N and the square of time t (in years). 8 units are presently produced. in 1 year, 16 units are expected. What is the expected production after 2 years?

(20) A piece of furniture is worth \$3600. The rate of depreciation of the value V (in dollars) at the time t (in years) is proportional to the square root of the value. The piece of furniture is worth \$2500 one year later. How long will it take for the piece of furniture to be worth \$900?

(21) The rate of increase of a population size P in a city is proportional to the product of the population size and the square root of time t (in years). Initially, the population is 100 000, and after 3 years it is 300 000. How long will it take for the population to be 1 200 000?

(22) The rate of increase of a population size P in a town, where t is the number of years after 1975, is proportional to the population size. In 1975, the population size was 2500, and in 1977 it was 3000. In what year will the population reach 4320 people?

(23) The rate of decay at time t (in hours) of a radioactive substance N is proportional to the square of the amount of substance present. If 80% of the initial amount of radioactive substance remains after 3 hours, find the remaining amount of radioactive substance at time t if initially the amount was 15g.

(24) The rate of increase of the number of members N of a club is proportional to the square of time t (the number of years after 1982). In 1982, the club had 1700 members. By 1985, the membership has increased to 4400 members. What is the number of members in 1994?

(25) The rate of increase of a population size P in a city is proportional to the population size and inversely proportional to the time t (number of years after 1965). In 1966, the population was 800 000. The population grew by 6% after 9 years. What was the population in 1983?

(26) A company has their sales volume S (in millions of dollars) at time t (in years) increasing at a rate that is proportional to the square root of the sales volume. At the present time, the sales volume is 16 millions of dollars. The company predicts that in two years, the sales volume will be 25 millions of dollars. How many years will it take for the sales volume to reach 49 millions of dollars?

(27) A computer company has their sales volume S (in millions of dollars) increasing at a rate that is proportional to the product of the square of both the sales volume and the time t (in years). The company started with a sales volume of 100 millions of dollars. Three years later, the sales volume reached 120 millions of dollars. When will the sales volume reach 165 millions of dollars?

(28) A company has their production increasing at a rate that is proportional to the square of the number of units N, and inversely proportional to the time t (in days). The first day, 300 units are produced. In 3 days, 400 units are expected. What will be the production after 27 days?

(29) A car is bought for \$36 000. The rate of depreciation of the value V (in dollars) at time t (in years) is proportional to the present value. After 2 years, the vehicle is worth \$18 000. What is the value of the car 4 years later?

(30) A piece of machinery is worth \$2500. The rate of depreciation of the value V (in dollars) at time t (in years) is proportional to the square root of its value. The piece of machinery will be worth \$900 three years later. What is the value of the piece of machinery after 6 years?

(31) The production N of a company during time t (in years) is increasing at a rate that is proportional to the square of the number of units. Initially, the production is 100 units, and after 2 years, the production is 200 units. Find the number of unit produced N as a function of time t.

(32) The production N of a company during time t (in years) is increasing at a rate that is proportional to the square root of time t. Initially, the production is 50 units, and after 4 years, the production is 150 units. Find the number of unit produced N as a function of time t.

(33) The production N of a company during time t (in years) is increasing at a rate that is proportional to the square root of both the number of units N and time t. Initially, the production is 400 units, and after 1 year, the production is 1764 units. Find the number of unit produced N as a function of time t.

(34) The production N of a company during time t (in years) is increasing at a rate that is inversely proportional to the square root of time t. Initially, the production is 124 units, and after 1 year, the production is 136 units. Find the number of unit produced N as a function of time t.

(35) The production N of a company during time t (in years) is decreasing at a rate that is proportional to the square of the number of units, and inversely proportional to the time t. In the first year, the production is 70 units, and in the 8^{th} year, the production is 35 units. Find the number of unit produced N as a function of time t.

(36) The production N of a company during time t (in years) is increasing at a rate that is inversely proportional to the number of units, and proportional to time t. Initially, the production is 8 units, and after 2 years, the production is 10 units. Find the number of unit produced N as a function of time t.

ANSWERS:

(1) mid 2005

(2) 1 823 260 people

(3) 60 millions of \$

(4) 32 400 units

(5) \$20 480

(6) 3 years

(7) About 10.32 weeks

(8) About 1.33 years

(9) 10.8g

(10) $\sim$ 64.5 years

(11) $N(t) = 10 \cdot (0.8)^{t/2}$

(12) $\sim$ 18.64g

(13) 4 years

(14) ~ 1.73 years

(15) \$1355.93

(16) 579 633 persons

(17) 49 998 persons

(18) 7000 units

(19) 2048 units

(20) 3 years

(21) ~ 5.17 years

(22) 6 years

(23) $N(t) = \frac{180}{t+12}$

(24) 174 500 members

(25) 860 707 persons

(26) 6 years

(27) ≈ 4 years

(28) 1200 units

(29) \$9000

(30) \$100

(31) $N(t) = \frac{400}{4-t}$

(32) $N(t) = 12.5t^{3/2} + 50$

(33) $N(t) = \left(22t^{3/2} + 20\right)^2$

(34) $N(t) = 12\sqrt{t} + 124$

(35) $N(t) = \frac{70\ln(8)}{\ln(8t)}$

(36) $N(t) = \sqrt{9t^2 + 64}$