201-203-RE - Supplement H - Series

Find an expression for the n^{th} partial sum s_n of each of the following telescoping series, and use it to determine whether the series converges or diverges. If a series converges, find its sum.

(1) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$

(2) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k^2-k}$

(3) $\displaystyle\sum_{k=1}^{\infty} \frac{2}{k^2+4k+3}$

(4) $\displaystyle\sum_{k=1}^{\infty} \ln\left(\frac{k}{k+1}\right)$

(5) $\displaystyle\sum_{k=2}^{\infty} \left[\frac{1}{\ln k} - \frac{1}{\ln(k+1)}\right]$

(6) $\displaystyle\sum_{k=1}^{\infty} \left(e^{\frac{1}{k}} - e^{\frac{1}{k+1}}\right)$

(7) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{4k^2-1}$

(8) $\displaystyle\sum_{k=3}^{\infty} \frac{3}{k^2+k-2}$

(9) $\displaystyle\sum_{k=2}^{\infty} \left[\sin\left(\frac{\pi}{k}\right) - \sin\left(\frac{\pi}{k+1}\right)\right]$

Use the integral test to determine whether the following series converge or diverge.

(10) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{5k-2}$

(11) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3^n}$

(12) $\displaystyle\sum_{k=1}^{\infty} ke^{-k^2}$

(13) $\displaystyle\sum_{n=3}^{\infty} \frac{n^3}{n^4-16}$

(14) $\displaystyle\sum_{k=1}^{\infty} \frac{k}{(k^2+1)^{\frac{3}{2}}}$

(15) $\displaystyle\sum_{k=2}^{\infty} \frac{1}{k\sqrt{\ln(k)}}$

State whether each of the following series is a geometric series or a p-series, and determine whether the series converges or diverges. Where possible, also find the sum of the series.

(16) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{7^k}$

(17) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^7}$

(18) $\displaystyle\sum_{n=1}^{\infty} \sqrt{n}$

(19) $\displaystyle\sum_{k=1}^{\infty} 6^{k+1}$

(20) $\displaystyle\sum_{k=1}^{\infty} k^{-3/4}$

(21) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$

(22) $2 + \frac{2}{5} + \frac{2}{25} + \frac{2}{125} + \cdots$

(23) $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \cdots$

(24) $1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots$

(25) $1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \cdots$

(26) $1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \cdots$

Use the ratio test to determine whether the following series converge or diverges. If the ratio test is inconclusive, state this.

(27) $\displaystyle\sum_{k=0}^{\infty} \frac{k!}{4^k}$

(28) $\displaystyle\sum_{n=1}^{\infty} n\left(\frac{3}{4}\right)^n$

(29) $\displaystyle\sum_{n=1}^{\infty} n\left(\frac{4}{3}\right)^n$

(30) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k k^2}{5^{k+2}}$

(31) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$

(32) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$

(33) $\displaystyle\sum_{n=1}^{\infty} (n+1)5^{-n}$

(34) $\displaystyle\sum_{k=1}^{\infty} \frac{2k!}{k^4}$

(35) $\displaystyle\sum_{k=1}^{\infty} \frac{5^k}{2^k+3}$

(36) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{4^n+1}$

(37) $\displaystyle\sum_{k=1}^{\infty} \frac{k^3}{3^k}$

(38) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n+1}}$

Determine whether the following series converge or diverges. Justify your answers by referencing a test, and showing why that test may be applied. **In the case of a convergent geometric or telescoping series, find the sum of the series.**

(39) $\displaystyle\sum_{k=2}^{\infty} \frac{5}{\sqrt[4]{k^3}}$

(40) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{6}{7}\right)^{n+1}$

(41) $\displaystyle\sum_{k=1}^{\infty} \ln(k)$

(42) $\displaystyle\sum_{n=5}^{\infty} \frac{4^{n-1}}{3n!}$

(43) $\displaystyle\sum_{n=4}^{\infty} \frac{36}{n^2-n-2}$

(44) $\displaystyle\sum_{n=0}^{\infty} \frac{(-2)^n}{3^{n+1}}$

(45) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n^2}$

(46) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n+1}-\frac{1}{n}\right)$

(47) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n^3}-\frac{1}{n^4}\right)$

(48) $\displaystyle\sum_{n=0}^{\infty} n(0.7)^n$

(49) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{5+3^{n+1}}$

(50) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{0.4}}$

(51) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(0.4)^n}$

(52) $\displaystyle\sum_{n=2}^{\infty} 3^{1+n}5^{1-n}$

(53) $\displaystyle\sum_{n=5}^{\infty} \frac{(-1)^n 2^n}{n^3}$

(54) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{\sqrt{n^6+1}}$

(55) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2+1}{(n+2)!}$

(56) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{1}{4}\right)^k$

(57) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{1}{3^k}-\frac{1}{3^{k+1}}\right)$

(58) $\displaystyle\sum_{k=3}^{\infty} \frac{4}{k^2+5k+6}$

(59) $\displaystyle\sum_{k=1}^{\infty} \frac{k}{4^k}$

(60) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{k}{100}\right)^k$

(61) $\displaystyle\sum_{k=2}^{\infty} \left(\frac{-3}{4}\right)^k$

(62) $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{n+1}$

(63) $\displaystyle\sum_{k=1}^{\infty} \frac{4^{k+2}}{3^{2k-1}}$

(64) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+n+1}{2n^2-6}$

(65) $\displaystyle\sum_{k=1}^{\infty} \frac{7}{3^k+2}$

(66) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$

(67) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{1}{3^k}+\frac{1}{k^3}\right)$

(68) $\displaystyle\sum_{k=1}^{\infty} \frac{(k!)^2}{(2k)!}$

(69) $\displaystyle\sum_{k=1}^{\infty} \frac{(k+3)!}{3!k!4^k}$

(70) $\displaystyle\sum_{k=1}^{\infty} \frac{k^2}{k!}$

(71) $\displaystyle\sum_{n=1}^{\infty} \frac{2\cdot 4\cdot 6\cdots(2n)}{1\cdot 4\cdot 7\cdots(3n-2)}$

(72) $\displaystyle\sum_{k=2}^{\infty} k^{-\frac{2}{3}}$

(73) $\displaystyle\sum_{k=2}^{\infty} \ln\left(1-\frac{1}{k^2}\right)$

ANSWERS:

(1) $s_n = 1 - \frac{1}{n+1}$; converges to 1

(2) $s_n = 1 - \frac{1}{n}$; converges to 1

(3) $s_n = \frac{5}{6} - \frac{1}{n+2} - \frac{1}{n+3}$; converges to $\frac{5}{6}$

(4) $s_n = -\ln(n+1)$; diverges

(5) $s_n = \frac{1}{\ln 2} - \frac{1}{\ln(n+1)}$; converges to $\frac{1}{\ln 2}$

(6) $s_n = e - e^{\frac{1}{n+1}}$; converges to $e - 1$

(7) $s_n = \frac{1}{2} - \frac{1}{4n+2}$; converges to $\frac{1}{2}$

(8) $s_n = \frac{13}{12} - \frac{1}{n} - \frac{1}{n+1} - \frac{1}{n+2}$; converges to $\frac{13}{12}$

(9) $s_n = 1 - \sin\left(\frac{\pi}{n+1}\right)$; converges to 1

(10) diverges since $\int_1^\infty \frac{dx}{5x-2} = \infty$

(11) converges since $\int_1^\infty \frac{dx}{3^x} = \frac{1}{3\ln 3}$

(12) converges since $\int_1^\infty xe^{-x^2}\ dx = \frac{1}{2e}$

(13) diverges since $\int_3^\infty \frac{n^3}{n^4-16}\ dx = \infty$

(14) converges since $\int_1^\infty \frac{x}{(x^2+1)^{3/2}}\ dx = \frac{1}{\sqrt{2}}$

(15) diverges since $\int_2^\infty \frac{dx}{x\sqrt{\ln x}} = \infty$

(16) geometric series with $r = \frac{1}{7}$; converges to $\frac{1}{6}$

(17) p-series with $p = 7$; converges

(18) p-series with $p = -\frac{1}{2}$; diverges

(19) geometric series with $r = 6$; diverges

(20) p-series with $p = \frac{3}{4}$; diverges

(21) p-series with $p = \frac{3}{2}$; diverges

(22) geometric series with $r = \frac{1}{5}$; converges to $\frac{5}{2}$

(23) p-series with $p = 2$; converges

(24) geometric series with $r = \frac{1}{4}$; converges to $\frac{4}{3}$

(25) geometric series with $r = \frac{2}{3}$; converges to 3

(26) p-series with $p = \frac{1}{2}$; diverges

(27) diverges

(28) converges

(29) diverges

(30) converges

(31) converges

(32) inconclusive

(33) converges

(34) diverges

(35) diverges

(36) diverges

(37) converges

(38) inconclusive

(39) diverges by p-series

(40) converges to 6 by geometric series

(41) diverges by test for divergence

(42) converges by ratio test

(43) converges to 13 by telescoping

(44) converges to $\frac{1}{5}$ by geometric series

(45) diverges by test for divergence or ratio test

(46) converges to -1 by telescoping

(47) converges by p-series

(48) converges by ratio test

(49) converges by ratio test

(50) diverges by p-series

(51) diverges by geometric series

(52) converges to $\frac{27}{2}$ by geometric series

(53) diverges by ratio test

(54) diverges by test for divergence

(55) converges by ratio test

(56) converges to $\frac{1}{3}$ by geometric series

(57) converges to $\frac{1}{3}$ by geometric series

(58) converges to $\frac{4}{5}$ by telescoping series

(59) converges by ratio test

(60) diverges by divergence test

(61) converges to $\frac{9}{28}$ by geometric series

(62) diverges by test for divergence

(63) converges to $\frac{192}{5}$ by geometric series

(64) diverges by test for divergence

(65) converges by ratio test

(66) diverges by p-series

(67) converges by geometric series and p-series

(68) converges by ratio test

(69) converges by ratio test

(70) converges by ratio test

(71) converges by ratio test

(72) diverges by p-series

(73) converges to $\ln(\frac{1}{2})$ by telescoping series