

## 201-203-RE - Supplement G - Applications of Geometric Series

- (1) A deposit of 35 dollars is made at the beginning of each month, for a period of 4 years, in an account that pays 1.5% interest, compounded monthly. Find the balance in the account at the end of the 4 years.
- (2) The annual net profit for a company, from 1995 to 2005, can be approximated by the model  $a_n = 10e^{0.2n}$ ,  $n = 1, 2, 3, \dots, N$ , where  $a_n$  is the annual net profit in millions of dollars, and  $n$  represents the year, with  $n = 1$  corresponding to 1995. Estimate the total net profit during this period.
- (3) A deposit of 15 dollars is made at the beginning of each month, for a period of 5 years, in an account that pays 0.9% interest, compounded monthly. Find the balance in the account at the end of the 5 years.
- (4) A deposit of 20 dollars is made every 3 months, for a period of 10 years, in an account that pays 1% interest, compounded quarterly. Find the balance in the account at the end of the 10 years.
- (5) A deposit of \$500 can be made with two options: earn 2% interest, compounded quarterly for 3 years, or earn 1.5%, compounded monthly for 3 years. Find the balance after 3 years of both options.
- (6) A deposit of \$1200 can be made with two options: earn 1.6% interest, compounded every 2 months for 5 years, or earn 2.3% interest, compounded semi-annually for 5 years. Find the balance after 5 years of both options.
- (7) The annual net profit for a company, from 1995 to 2000, can be approximated by the model  $a_n = 5e^{0.1n}$ , with  $n = 1, 2, 3, \dots, N$ , where  $a_n$  is the annual net profit (in millions of dollars), and  $n$  represents the year, with  $n = 1$  corresponding to 1995. Estimate the total net profit earned during this period.
- (8) To create a scholarship of \$600 that is to be awarded every year, we can use the series  $\sum_{n=1}^{+\infty} 600e^{-0.05n}$  to determine the sum of money that has to be deposited in an account that earns 5% interest, compounded continuously. Find the amount of money that must be deposited.
- (9) To create a scholarship of \$1200 that is to be awarded every year, we can use the series  $\sum_{n=1}^{+\infty} 1200e^{-0.03n}$  to determine the sum of money that has to be deposited in an account that earns 3% interest, compounded continuously. Find the amount of money that must be deposited.
- (10) A patient is given 50mg of a drug daily, for a long period of time. The amount of drug left in the patient's body after  $n$  years is given by  $\sum_{k=1}^n 50e^{-k/2}$ . Find the amount of drug in the patient's system after 4 years and after 15 years.
- (11) A deposit of 10 dollars is made at the beginning of each month, for a period of 6 years, into an account that pays 1% yearly interest, compounded 12 times per year. Find the balance in the account at the end of the 6 years.
- (12) The annual net profit for a company, from 1997 to 2003, can be approximated by the model  $a_n = 15e^{0.3n}$ , with  $n = 1, 2, 3, \dots, N$ , where  $a_n$  is the annual net profit (in millions of dollars), and  $n$  represents the year, with  $n = 1$  corresponding to 1997. Estimate the total net profit earned during this period.
- (13) A deposit of 30 dollars is made at the beginning of each month, for a period of 7 years, into an account that pays 1.1% yearly interest, compounded 12 times per year. Find the balance in the account at the end of the 7 years.

- (14) The annual net profit for a company, from 1996 to 2001, can be approximated by the model  $a_n = 6e^{0.05n}$ , with  $n = 1, 2, 3, \dots, N$ , where  $a_n$  is the annual net profit (in millions of dollars), and  $n$  represents the year, with  $n = 1$  corresponding to 1996. Estimate the total net profit earned during this period.
- (15) A deposit of 50 dollars is made bi-weekly, for a period of 8 years, into an account that pays 1.3% yearly interest, compounded every two weeks. Find the balance in the account at the end of the 8 years.
- (16) A deposit of \$700 can be made with two options: account  $A$  earns 1.8% yearly interest, compounded every two weeks, and account  $B$  earns 2.1% yearly interest, compounded monthly. The amount will stay in the account for a period of 4 years. What would the balance be after the 4 years, for both accounts?
- (17) A deposit of \$1500 can be made with two options: account  $A$  earns 1.4% yearly interest, compounded quarterly, and account  $B$  earns 1.9% yearly interest, compounded monthly. The amount will stay in the account for a period of 6 years. What would the balance be after the 6 years, for both accounts?
- (18) To create a scholarship of \$750 that is to be awarded every year, we can use the series  $\sum_{n=1}^{+\infty} 750e^{-0.04n}$  to determine the sum of money that has to be deposited in an account that earns 4% interest, compounded continuously. Find the amount of money that must be deposited.
- (19) To create a scholarship of \$1500 that is to be awarded every year, we can use the series  $\sum_{n=1}^{+\infty} 1500e^{-0.035n}$  to determine the sum of money that has to be deposited in an account that earns 3.5% interest, compounded continuously. Find the amount of money that must be deposited.
- (20) A patient is given 40mg of a drug daily, for a long period of time. The amount of drug left in the patient's body after  $n$  years is given by  $\sum_{k=1}^n 40e^{-k/5}$ . Find the amount of drug in the patient's system after 6 years and after 20 years.
- (21) Valery would like to save \$1 400 000 for her retirement. Her account has an annual interest rate of 2.7%, compounded semi-annually. If Valery wants to retire in 40 years, how much should she be saving semi-annually?
- (22) Suzana would like to save a \$40 000 down payment for a house. If she can invest at 1.6% interest compounded weekly over ten years, how much should she save each week?
- (23) Jennifer wants to save \$15 000 for a new car in eight years. If she can save at 1.1% interest compounded monthly, what should be her monthly deposit?

In problems 24 - 37, geometric series to write the following repeating decimals as fractions.

- (24)  $4.\overline{13}$       (26)  $3.\overline{09}$       (28)  $5.\overline{011}$       (30)  $7.\overline{34}$       (32)  $1.\overline{06}$       (34)  $8.\overline{01}$       (36)  $5.\overline{25}$   
 (25)  $6.\overline{04}$       (27)  $2.\overline{02}$       (29)  $10.\overline{23}$       (31)  $20.\overline{021}$       (33)  $2.\overline{03}$       (35)  $2.\overline{22}$       (37)  $12.\overline{302}$

**ANSWERS:**

- (1) \$1732.47                                      (3) \$920.89                                      (5) \$530.84 or \$523.00  
 (2) \$442.712 millions                              (4) \$842.36                                      (6) \$1299.81 or \$1345.36

(7) \$43.195 millions	(18) \$18 377.50	(29) 307/30
(8) \$11 702.50	(19) \$42 111.52	(30) 661/90
(9) \$39 403.00	(20) 126.25mg and 177.36mg	(31) 6607/330
(10) 66.64mg and 77.03mg	(21) \$9695.05	(32) 35/33
(11) \$742.34	(22) \$70.92	(33) 67/33
(12) \$414.738 millions	(23) \$149.41	(34) 793/99
(13) \$2620.71	(24) 409/99	(35) 20/9
(14) \$43.041 millions	(25) 598/99	(36) 520/99
(15) \$10 962.64	(26) 34/11	(37) 2768/225
(16) \$752.24 or \$761.28	(27) 200/99	
(17) \$1631.20 or \$1680.98	(28) 5006/999	