

201-203-RE - Supplement F - Series

Find an expression for the n^{th} partial sum s_n of each of the following telescoping series, and use it to determine whether the series converges or diverges. If a series converges, find its sum.

$$(1) \sum_{k=1}^{\infty} \frac{1}{k(k+1)}$$

$$(4) \sum_{k=1}^{\infty} \ln\left(\frac{k}{k+1}\right)$$

$$(7) \sum_{k=1}^{\infty} \frac{1}{4k^2 - 1}$$

$$(2) \sum_{k=2}^{\infty} \frac{1}{k^2 - k}$$

$$(5) \sum_{k=2}^{\infty} \left[\frac{1}{\ln k} - \frac{1}{\ln(k+1)} \right]$$

$$(8) \sum_{k=3}^{\infty} \frac{3}{k^2 + k - 2}$$

$$(3) \sum_{k=1}^{\infty} \frac{2}{k^2 + 4k + 3}$$

$$(6) \sum_{k=1}^{\infty} \left(e^{\frac{1}{k}} - e^{\frac{1}{k+1}} \right)$$

$$(9) \sum_{k=2}^{\infty} \left[\sin\left(\frac{\pi}{k}\right) - \sin\left(\frac{\pi}{k+1}\right) \right]$$

Use the integral test to determine whether the following series converge or diverge.

$$(10) \sum_{k=1}^{\infty} \frac{1}{5k - 2}$$

$$(12) \sum_{k=1}^{\infty} k e^{-k^2}$$

$$(14) \sum_{k=1}^{\infty} \frac{k}{(k^2 + 1)^{\frac{3}{2}}}$$

$$(11) \sum_{n=1}^{\infty} \frac{1}{3^n}$$

$$(13) \sum_{n=3}^{\infty} \frac{n^3}{n^4 - 16}$$

$$(15) \sum_{k=2}^{\infty} \frac{1}{k\sqrt{\ln(k)}}$$

State whether each of the following series is a geometric series or a p -series, and determine whether the series converges or diverges. Where possible, also find the sum of the series.

$$(16) \sum_{k=1}^{\infty} \frac{1}{7^k}$$

$$(19) \sum_{k=1}^{\infty} 6^{k+1}$$

$$(23) 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \dots$$

$$(17) \sum_{k=1}^{\infty} \frac{1}{k^7}$$

$$(20) \sum_{k=1}^{\infty} k^{-3/4}$$

$$(24) 1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots$$

$$(21) \sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$$

$$(25) 1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \dots$$

$$(18) \sum_{n=1}^{\infty} \sqrt{n}$$

$$(22) 2 + \frac{2}{5} + \frac{2}{25} + \frac{2}{125} + \dots$$

$$(26) 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \dots$$

Use the ratio test to determine whether the following series converge or diverge. If the ratio test is inconclusive, state this.

$$(27) \sum_{k=0}^{\infty} \frac{k!}{4^k}$$

$$(31) \sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$$

$$(35) \sum_{k=1}^{\infty} \frac{5^k}{2^k + 3}$$

$$(28) \sum_{n=1}^{\infty} n \left(\frac{3}{4} \right)^n$$

$$(32) \sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$$

$$(36) \sum_{n=1}^{\infty} \frac{n!}{4^n + 1}$$

$$(29) \sum_{n=1}^{\infty} n \left(\frac{4}{3} \right)^n$$

$$(33) \sum_{n=1}^{\infty} (n+1)5^{-n}$$

$$(37) \sum_{k=1}^{\infty} \frac{k^3}{3^k}$$

$$(30) \sum_{k=1}^{\infty} \frac{(-1)^k k^2}{5^{k+2}}$$

$$(34) \sum_{k=1}^{\infty} \frac{2k!}{k^4}$$

$$(38) \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n+1}}$$

Determine whether the following series converge or diverge. Justify your answers by referencing a test, and showing why that test may be applied. **In the case of a convergent geometric or telescoping series, find the sum of the series.**

$$(39) \sum_{k=2}^{\infty} \frac{5}{\sqrt[4]{k^3}}$$

$$(40) \sum_{n=0}^{\infty} \left(\frac{6}{7}\right)^{n+1}$$

$$(41) \sum_{k=1}^{\infty} \ln(k)$$

$$(42) \sum_{n=5}^{\infty} \frac{4^{n-1}}{3n!}$$

$$(43) \sum_{n=4}^{\infty} \frac{36}{n^2 - n - 2}$$

$$(44) \sum_{n=0}^{\infty} \frac{(-2)^n}{3^{n+1}}$$

$$(45) \sum_{n=1}^{\infty} \frac{2^n}{n^2}$$

$$(46) \sum_{n=1}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n}\right)$$

$$(47) \sum_{n=1}^{\infty} \left(\frac{1}{n^3} - \frac{1}{n^4}\right)$$

$$(48) \sum_{n=0}^{\infty} n(0.7)^n$$

$$(49) \sum_{n=1}^{\infty} \frac{2^n}{5 + 3^{n+1}}$$

$$(50) \sum_{n=1}^{\infty} \frac{1}{n^{0.4}}$$

$$(51) \sum_{n=1}^{\infty} \frac{1}{(0.4)^n}$$

$$(52) \sum_{n=2}^{\infty} 3^{1+n} 5^{1-n}$$

$$(53) \sum_{n=5}^{\infty} \frac{(-1)^n 2^n}{n^3}$$

$$(54) \sum_{n=1}^{\infty} \frac{n^3}{\sqrt{n^6 + 1}}$$

$$(55) \sum_{n=0}^{\infty} \frac{n^2 + 1}{(n+2)!}$$

$$(56) \sum_{k=1}^{\infty} \left(\frac{1}{4}\right)^k$$

$$(57) \sum_{k=1}^{\infty} \left(\frac{1}{3^k} - \frac{1}{3^{k+1}}\right)$$

$$(58) \sum_{k=3}^{\infty} \frac{4}{k^2 + 5k + 6}$$

$$(59) \sum_{k=1}^{\infty} \frac{k}{4^k}$$

$$(60) \sum_{k=1}^{\infty} \left(\frac{k}{100}\right)^k$$

$$(61) \sum_{k=2}^{\infty} \left(\frac{-3}{4}\right)^k$$

$$(62) \sum_{n=1}^{\infty} \frac{n-1}{n+1}$$

$$(63) \sum_{k=1}^{\infty} \frac{4^{k+2}}{3^{2k-1}}$$

$$(64) \sum_{n=1}^{\infty} \frac{n^2 + n + 1}{2n^2 - 6}$$

$$(65) \sum_{k=1}^{\infty} \frac{7}{3^k + 2}$$

$$(66) \sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$$

$$(67) \sum_{k=1}^{\infty} \left(\frac{1}{3^k} + \frac{1}{k^3}\right)$$

$$(68) \sum_{k=1}^{\infty} \frac{(k!)^2}{(2k)!}$$

$$(69) \sum_{k=1}^{\infty} \frac{(k+3)!}{3!k!4^k}$$

$$(70) \sum_{k=1}^{\infty} \frac{k^2}{k!}$$

$$(71) \sum_{n=1}^{\infty} \frac{2 \cdot 4 \cdot 6 \cdots (2n)}{1 \cdot 4 \cdot 7 \cdots (3n-2)}$$

$$(72) \sum_{k=2}^{\infty} k^{-\frac{2}{3}}$$

$$(73) \sum_{k=2}^{\infty} \ln\left(1 - \frac{1}{k^2}\right)$$

ANSWERS:

- (1) $s_n = 1 - \frac{1}{n+1}$; converges to 1
- (2) $s_n = 1 - \frac{1}{n}$; converges to 1
- (3) $s_n = \frac{5}{6} - \frac{1}{n+2} - \frac{1}{n+3}$; converges to $\frac{5}{6}$
- (4) $s_n = -\ln(n+1)$; diverges
- (5) $s_n = \frac{1}{\ln 2} - \frac{1}{\ln(n+1)}$; converges to $\frac{1}{\ln 2}$
- (6) $s_n = e - e^{\frac{1}{n+1}}$; converges to $e - 1$
- (7) $s_n = \frac{1}{2} - \frac{1}{4n+2}$; converges to $\frac{1}{2}$
- (8) $s_n = \frac{13}{12} - \frac{1}{n} - \frac{1}{n+1} - \frac{1}{n+2}$; converges to $\frac{13}{12}$
- (9) $s_n = 1 - \sin\left(\frac{\pi}{n+1}\right)$; converges to 1
- (10) diverges since $\int_1^\infty \frac{dx}{5x-2} = \infty$
- (11) converges since $\int_1^\infty \frac{dx}{3^x} = \frac{1}{3 \ln 3}$
- (12) converges since $\int_1^\infty x e^{-x^2} dx = \frac{1}{2e}$
- (13) diverges since $\int_3^\infty \frac{n^3}{n^4-16} dx = \infty$
- (14) converges since $\int_1^\infty \frac{x}{(x^2+1)^{3/2}} dx = \frac{1}{\sqrt{2}}$
- (15) diverges since $\int_2^\infty \frac{dx}{x\sqrt{\ln x}} = \infty$
- (16) geometric series with $r = \frac{1}{7}$; converges to $\frac{1}{6}$
- (17) p-series with $p = 7$; converges
- (18) p-series with $p = -\frac{1}{2}$; diverges
- (19) geometric series with $r = 6$; diverges
- (20) p-series with $p = \frac{3}{4}$; diverges
- (21) p-series with $p = \frac{3}{2}$; diverges
- (22) geometric series with $r = \frac{1}{5}$; converges to $\frac{5}{2}$
- (23) p-series with $p = 2$; converges
- (24) geometric series with $r = \frac{1}{4}$; converges to $\frac{4}{3}$
- (25) geometric series with $r = \frac{2}{3}$; converges to 3
- (26) p-series with $p = \frac{1}{2}$; diverges
- (27) diverges
- (28) converges
- (29) diverges
- (30) converges
- (31) converges
- (32) inconclusive
- (33) converges
- (34) diverges
- (35) diverges
- (36) diverges
- (37) converges
- (38) inconclusive
- (39) diverges by p-series
- (40) converges to 6 by geometric series
- (41) diverges by test for divergence
- (42) converges by ratio test
- (43) converges to 13 by telescoping
- (44) converges to $\frac{1}{5}$ by geometric series
- (45) diverges by test for divergence or ratio test
- (46) converges to -1 by telescoping

- (47) converges by p-series
- (48) converges by ratio test
- (49) converges by ratio test
- (50) diverges by p-series
- (51) diverges by geometric series
- (52) converges to $\frac{27}{2}$ by geometric series
- (53) diverges by ratio test
- (54) diverges by test for divergence
- (55) converges by ratio test
- (56) converges to $\frac{1}{3}$ by geometric series
- (57) converges to $\frac{1}{3}$ by geometric series
- (58) converges to $\frac{4}{5}$ by telescoping series
- (59) converges by ratio test
- (60) diverges by divergence test
- (61) converges to $\frac{9}{28}$ by geometric series
- (62) diverges by test for divergence
- (63) converges to $\frac{192}{5}$ by geometric series
- (64) diverges by test for divergence
- (65) converges by ratio test
- (66) diverges by p-series
- (67) converges by geometric series and p-series
- (68) converges by ratio test
- (69) converges by ratio test
- (70) converges by ratio test
- (71) converges by ratio test
- (72) diverges by p-series
- (73) converges to $\ln(\frac{1}{2})$ by telescoping series