1. Let $A(7,1,-1)$, $B(4,-2,-1)$, $C(3,0,-5)$, and $D(-6,-9,-5)$ be points in 3-space.
   (a) Find the vector $\overrightarrow{AB}$.
   (b) Find a vector equation for the line L that passes through C and is parallel to $\overrightarrow{AB}$.
   (c) Is the point D on the line L?

2. Let $A(5,-2,1)$, $B(0,-3,4)$, $C(1,-1,2)$, and $D(7,2,1)$ be points in 3-space.
   (a) Find the magnitude of the vector $\overrightarrow{AB}$.
   (b) Find the unit vector in the same direction as $\overrightarrow{AB}$.
   (c) Find *parametric equations* for the line L that passes through C and is parallel to $\overrightarrow{AB}$.
   (d) Is the point D on the line L? Justify your answer.

3. Let $A(3,0,-2)$, $B(5,1,-3)$, $C(-1,-2,0)$, and $D(-2,3,-9)$ be points.
   (a) Find *parametric equations* for the line L that passes through A and B.
   (b) Determine if C is on the line containing A and B. Justify your answer.
   (c) Show that triangle ABD is a right triangle.

4. Let $A(-3,6,1)$, $B(4,1,-1)$, and $C(7,-2,-3)$.
   (a) Find $\|\overrightarrow{BC}\|$.
   (b) Find the unit vector in the opposite direction of $\overrightarrow{BC}$.
   (c) Find *parametric equations* for the line L that passes through A and is parallel to $\overrightarrow{BC}$.
   (d) Find a point on the line L other that A.
   (e) Find a *vector equation* for the plane containing the three points A, B, and C.

5. Let L be the line with vector equation

   $(x,y,z) = (1,2,3) + t(4,5,6)$.

   For each equation below, determine if it is also a vector equation for the line L.
   (a) $(x,y,z) = (2,4,6) + t(8,10,12)$
   (b) $(x,y,z) = (-3,-3,-3) + t(8,10,12)$
   (c) $(x,y,z) = (-3,-3,-3) + t(4,5,6)$
   (d) $(x,y,z) = (1,2,3) + t(3,3,3)$

6. Find an equation of the line passing through $(1,-4)$ and $(3,7)$:
   (a) in standard form $Ax + By = C$;
   (b) in vector form;
   (c) in parametric form.

7. Find an equation of the line through $(1,-5)$ with slope$= -\frac{2}{3}$:
   (a) in standard form $Ax + By = C$;
   (b) in vector form;
   (c) in parametric form.

8. Find a vector equation of the line L which:
   (a) is parallel to $(2,-1,0)$ and passes through $P(1,-1,3)$.
   (b) passes through $P(3,-1,4)$ and $Q(1,0,-1)$.
   (c) is parallel to $(1,2,-7)$ and passes through $O(0,0,0)$.
   (d) passes through $P(1,0,-3)$ and parallel to the line
   $$\begin{cases} x = -1 + 2t \\ y = 2 - t \\ z = 3 + 3t \end{cases}$$
   (e) passes through $P(2,-1,1)$ and parallel to the line $(x,y,z) = (2,1,0) + t(-1,0,1)$

9. For each set of planes determine if the intersection is a plane, a line, a point, or the empty set $\emptyset$.
   (a) $\begin{cases} 20x + 30y - 30z = 5 \\ -16x - 24y + 24z = -4 \end{cases}$
   (b) $\begin{cases} 20x + 30y - 30z = 4 \\ -16x - 24y + 24z = -5 \end{cases}$
   (c) $\begin{cases} 20x + 30y - 20z = 5 \\ -16x - 24y + 24z = -4 \end{cases}$
   (d) $\begin{cases} x + y = 1 \\ x + 2y + 3z = 4 \\ 4x + 3y + 2z = 1 \end{cases}$
   (e) $\begin{cases} x + y + z = 1 \\ x + 2y + 3z = 4 \\ 4x + 3y + 2z = 1 \end{cases}$
   (f) $\begin{cases} x + y + z = 0 \\ x + 2y + 3z = 4 \\ 4x + 3y + 2z = 1 \end{cases}$

10. Find a standard equation $(ax + by = c)$ for the line in $\mathbb{R}^2$ that fits each description:
    (a) Through $(1,3)$ and perpendicular to $\langle 7,-2\rangle$.
    (b) Through $(1,3)$ and parallel to $\langle 7,-2\rangle$.
    (c) Through $(2,0)$ and $(0,9)$.

11. Find a standard equation $(ax + by + cz = d)$ of the plane in $\mathbb{R}^3$ that fits each description:
    (a) Through $(4,0,-1)$ and parallel to both $\langle 5,1,-1\rangle$ and $\langle -2,3,0\rangle$.
    (b) Through $(6,6,-2)$, $(1,6,1)$, and $(2,9,1)$
    (c) Through $(1,1,4)$, $(2,-3,1)$, and $(-1,5,2)$
    (d) Through $(2,1,1)$, $(3,2,3)$, and $(-2,-1,3)$

12. Consider the system: $\begin{cases} kx + y + z = 0 \\ x + y + 2z = 0 \\ x + 2y + z = 0 \end{cases}$

For what value(s) of k do the planes...

(a) intersect at a point?

(b) intersect in a line?

(c) have no points of intersection?

13. Consider the system: $\begin{cases} 2x - 4y + 6z = 2 \\ 3x - 6y + hz = k \end{cases}$

For what value(s) of h and k do the planes...

(a) intersect in a plane?

(b) intersect in a line?

(c) have no points of intersection?

(d) intersect at a right angle? (perpendicular)

ANSWERS:

1. (a) $\overrightarrow{AB} = (-3, -3, 0)$

(b) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -5 \end{bmatrix} + t \begin{bmatrix} -3 \\ -3 \\ 0 \end{bmatrix}$.

(c) Yes. D occurs when $t = 3$ in the vector equation above.

2. Let $A(5, -2, 1)$, $B(0, -3, 4)$, $C(1, -1, 2)$, and $D(7, 2, 1)$ be points in 3-space.

(a) $\|\overrightarrow{AB}\| = \sqrt{35}$.

(b) $\frac{1}{\sqrt{35}}(-5, -1, 3)$

(c) $\begin{cases} x = 1 - 5t \\ y = -1 - t \\ z = 2 + 3t \end{cases}$

(d) D is not on the line since $\begin{cases} 7 = 1 - 5t \\ 2 = -1 - t \\ 1 = 2 + 3t \end{cases}$ has no solution.

3. (a) $\begin{cases} x = 3 + 2t \\ y = t \\ z = -2 - t \end{cases}$

(b) Yes, when $t = -2$ in the equation above.

(c) Angle $A = 90^o$

4. Let $A(-3, 6, 1)$, $B(4, 1, -1)$, and $C(7, -2, -3)$.

(a) $\|\overrightarrow{BC}\| = \sqrt{22}$.

(b) $-\frac{1}{\sqrt{22}}(3, -3, -2)$

(c) $\begin{cases} x = -3 + 3t \\ y = 6 - 3t \\ z = 1 - 2t \end{cases}$

(d) In the equation above, $t = 1$ gives $(0, 3, -1)$ for example.

(e) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ -1 \end{bmatrix} + s \begin{bmatrix} 7 \\ -5 \\ 2 \end{bmatrix} + t \begin{bmatrix} 3 \\ -3 \\ -2 \end{bmatrix}$.

Note: Many answers possible, we chose one that uses $\overrightarrow{BC}$.

5. (a) No.

(b) Yes.

(c) Yes.

(d) No.

6. (a) $11x - 2y = 19$

(b) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \end{bmatrix} + t \begin{bmatrix} 2 \\ 11 \end{bmatrix}$.

OR $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix} + t \begin{bmatrix} 2 \\ 11 \end{bmatrix}$.

(c) $\begin{cases} x = 1 + 2t \\ y = -4 + 11t \end{cases}$ OR $\begin{cases} x = 3 + 2t \\ y = 7 + 11t \end{cases}$

7. (a) $2x + 3y = -13$

(b) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \end{bmatrix} + t \begin{bmatrix} 3 \\ -2 \end{bmatrix}$.

(c) $\begin{cases} x = 1 + 3t \\ y = -5 - 2t \end{cases}$

8. (a) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} + t \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$.

(b) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix} + t \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}$

OR $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + t \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}$.

(c) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = t \begin{bmatrix} 1 \\ 2 \\ -7 \end{bmatrix}$.

(d) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix} + t \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$.

(e) $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$.

9. (a) Plane.

   (b) $\emptyset$.

   (c) Line.

   (d) Point.

   (e) Line.

   (f) $\emptyset$.

10. (a) $7x - 2y = 1$

    (b) $2x + 7y = 23$

    (c) $9x + 2y = 18$

11. (a) $3x + 2y + 17z = -5$

    (b) $3x - y + 5z = 2$

    (c) $5x + 2y - z = 3$

    (d) $3x - 5y + z = 2$

12. (a) $k \neq 2/3$

    (b) $k = 2/3$

    (c) None.

13. (a) $h = 9, k = 3$

    (b) $h \neq 9$

    (c) $h = 9, k \neq 3$

    (d) $h = -5$