Make a concavity table for the following functions, and find the points of inflection.

(1) $f(x)=\dfrac{3x^3+10x-24}{2x}$ (2) $f(x)=-x^4+2x^3+5x$ (3) $f(x)=\dfrac{x^3-x^2-8}{x-1}$

(4) $f(x)=\dfrac{x^3+4x+27}{x}$ (5) $f(x)=x^4+4x^3-5x$ (6) $f(x)=\dfrac{3}{2}x^2+\dfrac{12}{x-1}$

(7) $f(x)=x^4+2x^3-12x^2$ (8) $f(x)=4x^5+5x^4-80x^3$

Find the local extrema of the following functions, using the second derivative test.

(9) $f(x)=\dfrac{1}{4}x^4-\dfrac{5}{3}x^3+2x^2+\dfrac{35}{3}$ (10) $f(x)=\dfrac{6x^3+96}{x}$ (11) $f(x)=\dfrac{10\left(x^2+x+4\right)}{x+1}$

(12) $f(x)=\dfrac{3}{4}x^4+5x^3+9x^2-\dfrac{15}{4}$ (13) $f(x)=\dfrac{x^3-54}{x}$ (14) $f(x)=\dfrac{-3\left(x^2+2x+4\right)}{x+2}$

(15) $f(x)=\dfrac{1}{4}x^4+x^3-\dfrac{1}{2}x^2-3x$

Sketch the graph of the following functions:

(16) $f(x)=\dfrac{(x-2)(2x-1)}{(x+1)^2}$ with $f'(x)=\dfrac{9(x-1)}{(x+1)^3}$ and $f''(x)=\dfrac{18\,(2-x)}{(x+1)^4}$

(17) $f(x)=\left(\dfrac{x+2}{x-2}\right)^2$ with $f'(x)=\dfrac{-8(x+2)}{(x-2)^3}$ and $f''(x)=\dfrac{16\,(x+4)}{(x-2)^4}$

(18) $f(x)=\dfrac{6x^2}{4-x^2}$ with $f'(x)=\dfrac{48x}{(4-x^2)^2}$ and $f''(x)=\dfrac{48\,(3x^2+4)}{(4-x^2)^3}$

(19) $f(x)=\dfrac{6}{x^2+4x}$ with $f'(x)=\dfrac{-12(x+2)}{(x^2+4x)^2}$ and $f''(x)=\dfrac{12\,(3x^2+12x+16)}{(x^2+4x)^3}$

(20) $f(x)=\dfrac{1}{5}x^5-\dfrac{2}{3}x^3+x$ with $f'(x)=(x+1)^2\,(x-1)^2$ and $f''(x)=4x\,(x+1)\,(x-1)$

(21) $f(x)=x^3+9x^2+120$ with $f'(x)=3x(x+6)$ and $f''(x)=6x+18$

(22) $f(x)=(x-1)^4\,(3x+2)$ with $f'(x)=5\,(x-1)^3\,(3x+1)$ and $f''(x)=60x\,(x-1)^2$

(23) $f(x)=x+\dfrac{1}{x+2}$ with $f'(x)=\dfrac{(x+1)(x+3)}{(x+2)^2}$ and $f''(x)=\dfrac{2}{(x+2)^3}$

(24) $f(x)=\dfrac{x^2}{x-1}$ with $f'(x)=\dfrac{x(x-2)}{(x-1)^2}$ and $f''(x)=\dfrac{2}{(x-1)^3}$

(25) $f(x)=\dfrac{(x-2)(3x+1)}{(x-1)^2}$ with $f'(x)=\dfrac{9-x}{(x-1)^3}$ and $f''(x)=\dfrac{2\,(x-13)}{(x-1)^4}$

(26) $f(x)=\left(\dfrac{x+3}{x+1}\right)^2$ with $f'(x)=\dfrac{-4(x+3)}{(x+1)^3}$ and $f''(x)=\dfrac{8\,(x+4)}{(x+1)^4}$

(27) $f(x)=\dfrac{2x^2}{x^2-1}$ with $f'(x)=\dfrac{-4x}{(x^2-1)^2}$ and $f''(x)=\dfrac{4\,(3x^2+1)}{(x^2-1)^3}$

(28) $f(x)=\dfrac{4}{x^2-4x}$ with $f'(x)=\dfrac{8(2-x)}{(x^2-4x)^2}$ and $f''(x)=\dfrac{8(3x^2-12x+16)}{(x^2-4x)^3}$

(29) $f(x)=\dfrac{1}{5}x^5-\dfrac{8}{3}x^3-9x$ with $f'(x)=\left(x^2-9\right)\left(x^2+1\right)$ and $f''(x)=4x\left(x^2-4\right)$

(30) $f(x)=-x^3+18x^2+200$ with $f'(x)=3x(12-x)$ and $f''(x)=36-6x$

(31) $f(x)=(x+2)^4\,(4-3x)$ with $f'(x)=5\,(x+2)^3\,(2-3x)$ and $f''(x)=-60x\,(x+2)^2$

(32) $f(x)=3x+\dfrac{3}{x+1}$ with $f'(x)=\dfrac{3x(x+2)}{(x+1)^2}$ and $f''(x)=\dfrac{6}{(x+1)^3}$

(33) $f(x)=\dfrac{x^2}{x+2}$ with $f'(x)=\dfrac{x(x+4)}{(x+2)^2}$ and $f''(x)=\dfrac{8}{(x+2)^3}$

Sketch a function $f(x)$ with the following requirements:

(34) Points at $(-3,2)$, $(-2,0)$, $(0,-2)$, $(1,0)$, $\lim_{x\to+\infty} f(x)=1$
for $x<-3$: $f'(x)<0$; $f''(x)<0$
for $-3<x<0$: $f'(x)<0$; $f''(x)>0$
for $x>0$: $f'(x)>0$; $f''(x)<0$

(35) Points at $(-3,0)$, $(-2,1)$, $(-1,0)$, $(0,-0.5)$, $(1,-2)$, $\lim_{x\to+\infty} f(x)=0$
$f'(x)<0$ for $-2<x<1$; $f'(x)>0$ for $x<-2$ or $x>1$
$f''(x)<0$ for $x<-2$ or $x>-1$; $f''(x)>0$ for $-2<x<-1$

(36) Points at $(-2,0)$, $(-1,-1)$, $(0,0)$ vertical asymptote at $x=1$ and $\lim_{x\to+\infty} f(x)=2$
for $x<-1$: $f'(x)<0$; $f''(x)<0$
for $-1<x<0$: $f'(x)>0$; $f''(x)<0$
for $0<x<1$: $f'(x)>0$; $f''(x)>0$
for $x>1$: $f'(x)<0$; $f''(x)>0$

(37) Points at $(-3,0)$, $(-1,-1)$, $(0,-2)$, $(1,-1)$ vertical asymptote at $x=-2$
$\lim_{x\to-\infty} f(x)=1$ $\lim_{x\to+\infty} f(x)=0$
$f'(x)<0$ for $x<-2$ or $-1<x<0$; $f'(x)>0$ for $-2<x<-1$ or $0<x<1$ or $x>1$
$f''(x)<0$ for $x<-2$ or $-2<x<-1$ or $x>1$; $f''(x)>0$ for $-1<x<1$

(38) Points at $(-2,2)$, $(0,1)$, $(2,2)$ and $\lim_{x\to-\infty} f(x)=0$
for $x<-2$: $f'(x)>0$; $f''(x)>0$
for $-2<x<0$: $f'(x)<0$; $f''(x)<0$
for $0<x<2$: $f'(x)>0$; $f''(x)<0$
for $x>2$: $f'(x)>0$; $f''(x)>0$

(39) Points at $(-2,1)$, $(0,-1)$, $(2,0)$ and $\lim_{x\to+\infty} f(x)=2$
$f'(x)<0$ for $x<0$; $f'(x)>0$ for $x>0$
$f''(x)<0$ for $-2<x<0$ or $x>2$; $f''(x)>0$ for $x<-2$ or $0<x<2$

(40) Points at $(-2,0)$, $(0,0)$ vertical asymptote at $x=-1$ and $\lim_{x\to+\infty} f(x)=1$

for $x<-2$: $f'(x)>0$; $f''(x)<0$
for $-2<x<-1$: $f'(x)<0$; $f''(x)<0$
for $-1<x<0$: $f'(x)<0$; $f''(x)>0$
for $x>0$: $f'(x)>0$; $f''(x)<0$

(41) Domain: $-3<x\leqslant 4$; Points at $(-1,0)$, $(0,-1)$, $(1,0)$, $(4,2)$
$f'(x)<0$ for $-3<x<0$; $f'(x)>0$ for $0<x<4$
$f''(x)<0$ for $-3<x<-1$ or $1<x<4$; $f''(x)>0$ for $-1<x<1$

(42) Domain: $-2\leqslant x<4$; Points at $(-2,-1)$, $(0,0)$, $(2,2)$
$f'(x)<0$ for $2<x<4$; $f'(x)>0$ for $-2<x<2$
$f''(x)<0$ for $-2<x<0$ or $2<x<4$; $f''(x)>0$ for $0<x<2$

(43) Domain: $-4<x\leqslant 3$; Points at $(0,1)$, $(1,0)$, $(3,2)$; vertical asymptote at $x=-2$
for $-4<x<-2$: $f'(x)>0$; $f''(x)>0$
for $-2<x<0$: $f'(x)<0$; $f''(x)>0$
for $0<x<1$: $f'(x)<0$; $f''(x)<0$
for $1<x<3$: $f'(x)>0$; $f''(x)<0$

ANSWERS:

(1)

x	$-\infty$		0		2		$+\infty$
$f''(x)$		$+$	$\vert$	$-$	0	$+$	
$f(x)$		$\cup$	$\vert$	$\cap$	$\vert$	$\cup$	

POI: $(2,5)$

(2)

x	$-\infty$		0		1		$+\infty$
$f''(x)$		$-$	0	$+$	0	$-$	
$f(x)$		$\cap$	$\vert$	$\cup$	$\vert$	$\cap$	

POI: $(0,0)$ $(1,6)$

(3)

x	$-\infty$		1		3		$+\infty$
$f''(x)$		$+$	$\vert$	$-$	0	$+$	
$f(x)$		$\cup$	$\vert$	$\cap$	$\vert$	$\cup$	

POI: $(3,5)$

(4)

x	$-\infty$		-3		0		$+\infty$
$f''(x)$		$+$	0	$-$	$\vert$	$+$	
$f(x)$		$\cup$	$\vert$	$\cap$	$\vert$	$\cup$	

POI: $(-3,4)$

(5)

x	$-\infty$		-2		0		$+\infty$
$f''(x)$		$-$	0	$+$	0	$-$	
$f(x)$		$\cap$	$\vert$	$\cup$	$\vert$	$\cap$	

POI: $(0,0)$ $(-2,-6)$

(6)

x	$-\infty$		-1		1		$+\infty$
$f''(x)$		$+$	0	$-$	$\vert$	$+$	
$f(x)$		$\cup$	$\vert$	$\cap$	$\vert$	$\cup$	

POI: $\left(-1,-\frac{9}{2}\right)$

(7)

x	$-\infty$		-2		1		$+\infty$
$f''(x)$		$+$	0	$-$	0	$+$	
$f(x)$		$\cup$	$\vert$	$\cap$	$\vert$	$\cup$	

POI: $(-2,-48)$ $(1,-9)$

(8)

x	$-\infty$		-2.86		0		2.10		$+\infty$
$f''(x)$		$-$	0	$+$	0	$-$	0	$+$	
$f(x)$		$\cap$	$\vert$	$\cup$	$\vert$	$\cap$	$\vert$	$\cup$	

POI: $(-2.86,1433.00)$ $(0,0)$ $(2.10,-481.74)$

(9) Local Min: $(0,11.66)$, $(4,1)$; Local Max: $(1,12.25)$ (10) Local Min: $(2,72)$

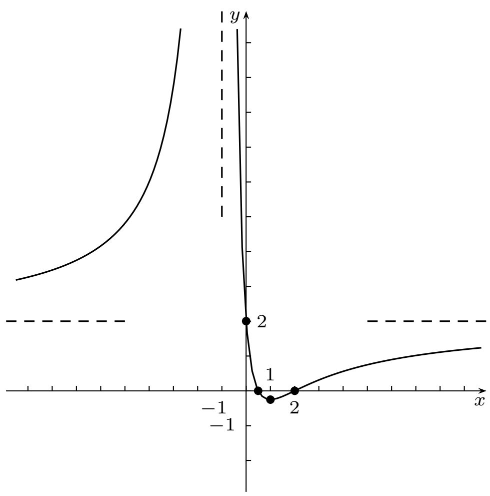
(11) Local Min: $(1, 30)$; Local Max: $(-3, 15)$

(12) Local Min: $(-3, 3)$, $(0, 0)$; Local Max: $(-2, 4.25)$ (13) Local Min: $(-3, 27)$

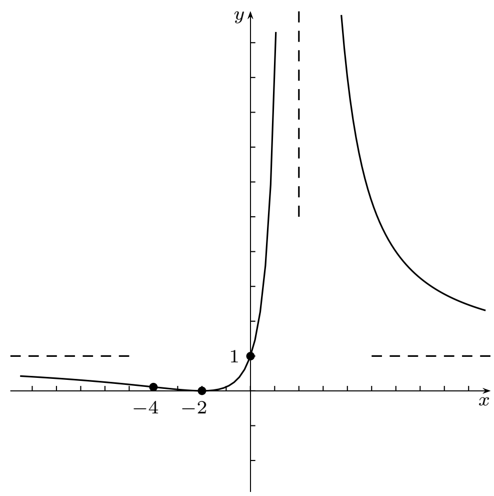
(14) Local Min: $(-4, 18)$; Local Max: $(0, -6)$

(15) Local Min: $(-3, -2.25)$, $(1, -2.25)$; Local Max: $(-1, 1.75)$

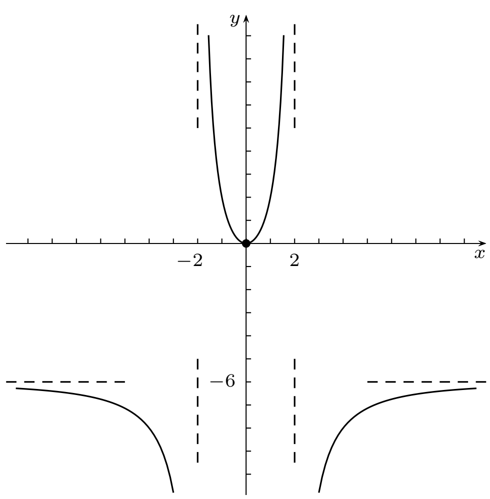
(16)



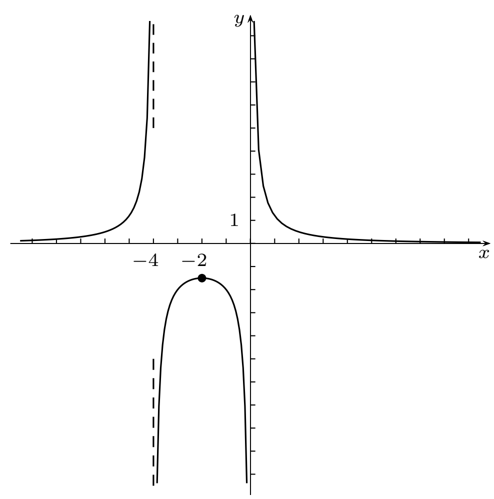
(17)



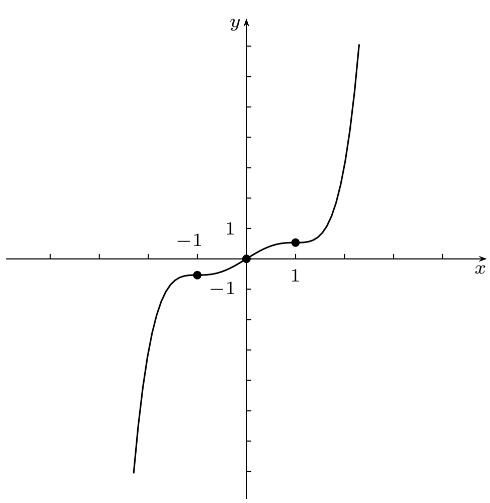
(18)



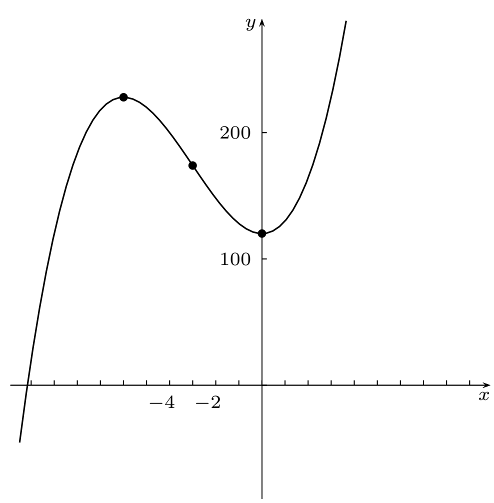
(19)



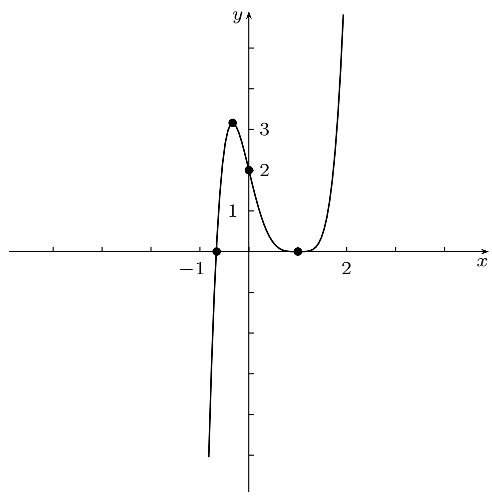
(20)



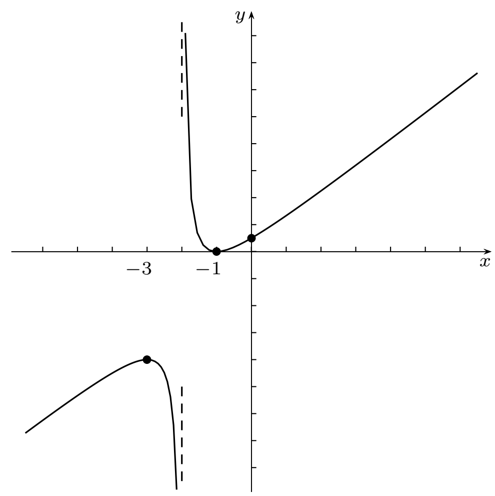
(21)



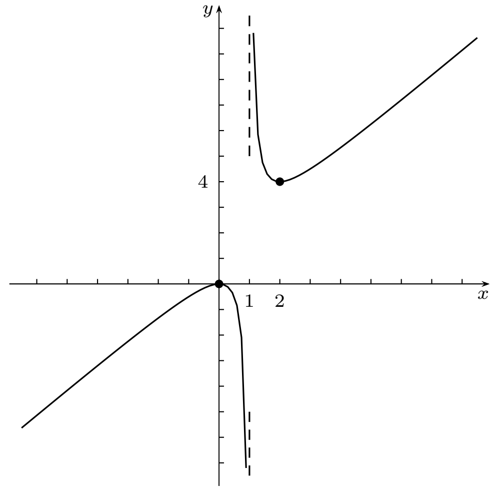
(22)



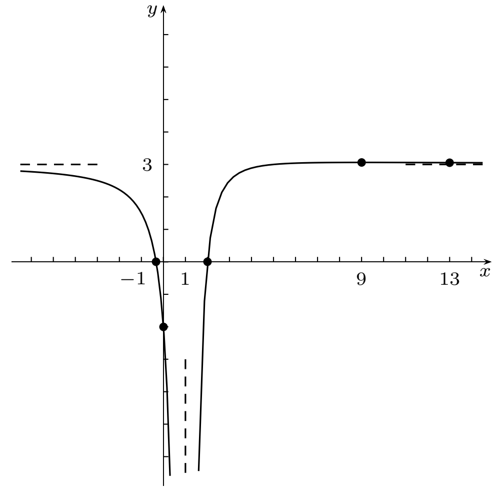
(23)



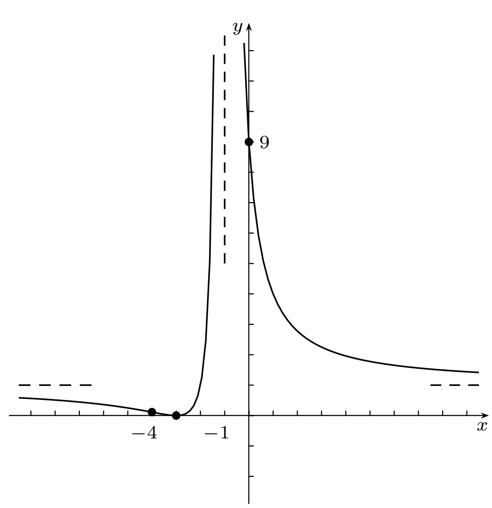
(24)



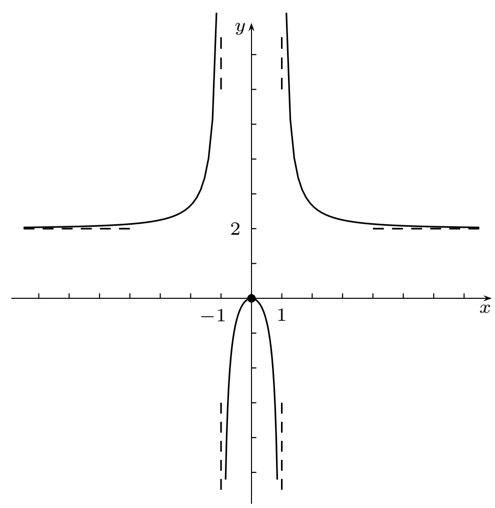
(25)



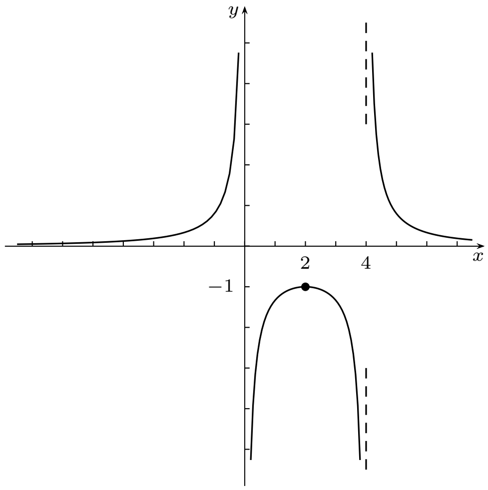
(26)



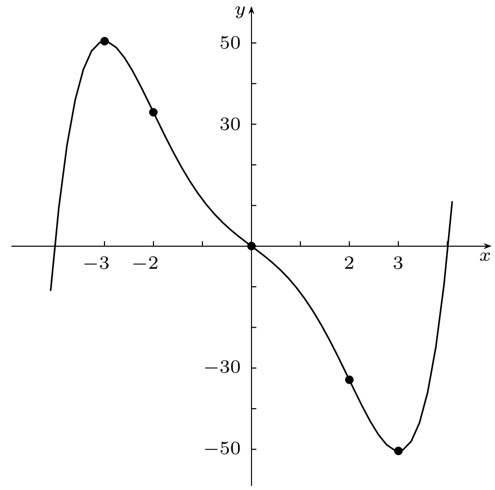
(27)



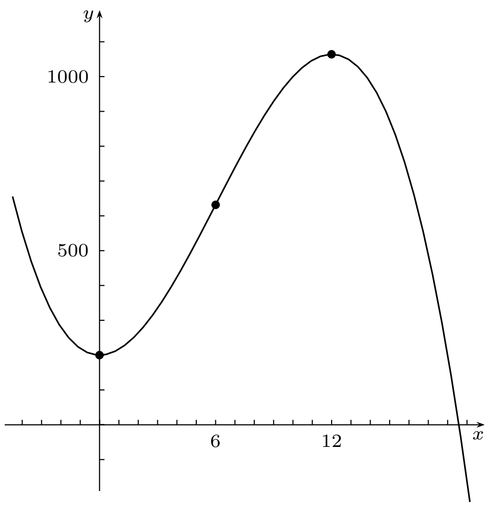
(28)



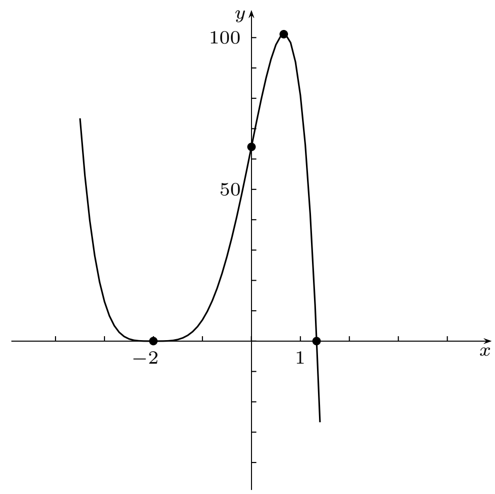
(29)



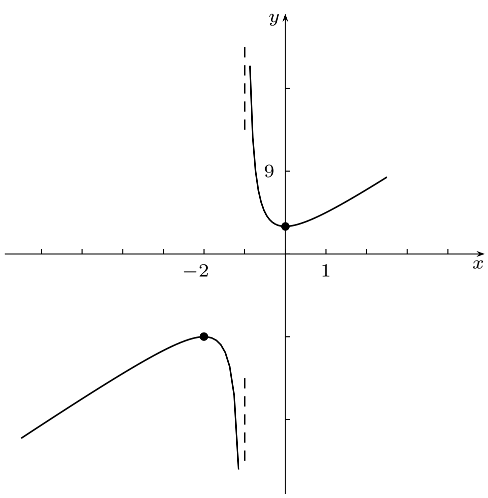
(30)



(31)



(32)



(33)