Find the discontinuities of the given functions. In each case, determine if the discontinuity is removable or non-removable. When the discontinuity is removable, how can the function be redefined so that it is continuous?

(1) $r(x) = \dfrac{x^2 + 2x - 3}{x^2 - 1}$

(2) $q(x) = \begin{cases} 4x - 1 & \text{if} \quad x \leqslant 1 \\ 2 - x^2 & \text{if} \quad x > 1 \end{cases}$

(3) $f(x) = \begin{cases} \sqrt{x} + 2 & \text{if} \quad x \geqslant 1 \\ 4 - x + x^2 & \text{if} \quad x < 1 \end{cases}$

(4) $g(x) = \dfrac{x + 2}{x^2 + x - 2}$

(5) $t(x) = \dfrac{3x^2}{6x + x^2}$

(6) $g(x) = \dfrac{x^2 - 2x - 3}{x^2 - 9}$

(7) $h(x) = \begin{cases} x^2 - 1 & \text{if} \quad x \leqslant -2 \\ 3x + 1 & \text{if} \quad x > -2 \end{cases}$

(8) $G(x) = \begin{cases} x^2 + x & \text{if} \quad x \leqslant -1 \\ x^3 & \text{if} \quad x > -1 \end{cases}$

(9) $h(x) = \dfrac{x - 3}{x^2 - 3x}$

(10) $f(x) = \dfrac{x^2}{4x - x^2}$

(11) $h(x) = \dfrac{4 - x}{x^2 - 7x + 12}$

(12) $f(x) = \begin{cases} 3x - 5 & \text{if} \quad x < 2 \\ \sqrt{x - 1} & \text{if} \quad x > 2 \end{cases}$

(13) $f(x) = \begin{cases} 5x - 1 & \text{if} \quad x > 1 \\ 2 & \text{if} \quad x = 1 \\ 3x^2 + 1 & \text{if} \quad x < 1 \end{cases}$

(14) $m(x) = \begin{cases} 11 - x^2 & \text{if} \quad x \geqslant -3 \\ |x + 1| & \text{if} \quad x < -3 \end{cases}$

(15) $s(x) = \dfrac{2 - x}{x^2 + 2x - 8}$

(16) $N(x) = \begin{cases} x^2 + 3 & \text{if} \quad x < -2 \\ -2x + 3 & \text{if} \quad x > -2 \end{cases}$

(17) $p(x) = \begin{cases} \sqrt{x + 6} & \text{if} \quad x > 3 \\ 6 & \text{if} \quad x = 3 \\ x^2 - 6 & \text{if} \quad x < 3 \end{cases}$

(18) $q(x) = \begin{cases} |x - 1| & \text{if} \quad x \leqslant -1 \\ 2x^2 & \text{if} \quad x > -1 \end{cases}$

(19) $g(x) = \dfrac{x^2 - 3x - 4}{x^2 - 1}$

(20) $f(x) = \begin{cases} \dfrac{2}{x - 1} & \text{if} \quad x < 2 \\ \sqrt{6 - x} & \text{if} \quad x \geqslant 2 \end{cases}$

(21) $g(x) = \begin{cases} \dfrac{3}{x^2-4} & \text{if} \quad x > 1 \\ x^2 - 1 & \text{if} \quad x \leqslant 1 \end{cases}$

(22) $\begin{cases} \dfrac{2x^2+11x+14}{x^2+7x+10} & \text{if} \quad x < -2 \\ \sqrt{x+3} & \text{if} \quad -2 \leqslant x \leqslant 6 \\ \dfrac{3x^2-13x-30}{x^2-10x+24} & \text{if} \quad x > 6 \end{cases}$

Find all the possible values of $k$ such that the following functions are continuous over $\mathbb{R}$.

(23) $f(x) = \begin{cases} 12 & \text{if} \quad x \leqslant -3 \\ kx+3 & \text{if} \quad -3 < x < 5 \\ -12 & \text{if} \quad x \geqslant 5 \end{cases}$

(24) $g(x) = \begin{cases} 3x - 4k & \text{if} \quad x \geqslant 5 \\ 2x + 9 & \text{if} \quad x < 5 \end{cases}$

(25) $h(x) = \begin{cases} -x^2 - 5k & \text{if} \quad x < 2 \\ k^2 - \dfrac{20}{x} & \text{if} \quad x \geqslant 2 \end{cases}$

(26) $f(x) = \begin{cases} \dfrac{x^2+2x-3}{x-1} & \text{if} \quad x \neq 1 \\ k^2 & \text{if} \quad x = 1 \end{cases}$

(27) $F(x) = \begin{cases} kx^2 + 2k^2x - 4 & \text{if} \quad x \leqslant 1 \\ 4kx^2 + k^2x + 6 & \text{if} \quad x > 1 \end{cases}$

(28) $G(x) = \begin{cases} x^2 + k^2x & \text{if} \quad x \leqslant 1 \\ 5k + 7x & \text{if} \quad x > 1 \end{cases}$

**ANSWERS:**

(1) NR: $x = -1$ R: $x = 1$, $f(1) = 2$ (2) NR: $x = 1$ (3) NR: $x = 1$
(4) NR: $x = 1$ R: $x = -2$, $f(-2) = \frac{-1}{3}$ (5) NR: $x = -6$ R: $x = 0$, $f(0) = 0$
(6) NR: $x = -3$ R: $x = 3$, $f(3) = \frac{2}{3}$ (7) NR: $x = -2$ (8) NR: $x = -1$
(9) NR: $x = 0$ R: $x = 3$, $f(3) = \frac{1}{3}$ (10) NR: $x = 4$ R: $x = 0$. $f(0) = 0$
(11) NR: $x = 3$ R: $x = 4$, $f(4) = -1$ (12) R: $x = 2$, $f(2) = 1$
(13) R: $x = 1$, $f(1) = 4$ (14) No discontinuity.
(15) NR: $x = -4$ R: $x = 2$, $f(2) = \frac{-1}{6}$ (16) R: $x = -2$, $f(-2) = 7$
(17) R: $x = 3$, $f(3) = 3$ (18) No discontinuity.
(19) NR: $x = 1$ R: $x = -1$, $f(-1) = \frac{5}{2}$ (20) NR: $x = 1$ Discontinuous if $x > 6$
(21) NR: $x = 1$ and $x = 2$ (22) NR: $x = -5$ and $x = 6$ (23) $k = -3$
(24) $k = -1$ (25) $k = -6$ or $k = 1$ (26) $k = \pm 2$ (27) $k = -2$ or $k = 5$
(28) $k = -1$ or $k = 6$