201-103-RE - Supplement F: Applications of the First Derivative

Find the intervals of increase and decrease, and and local extrema of the following functions:

(1) $f(x) = -\frac{x^3}{3} + x^2 + 3x + 4$

(2) $f(x) = \frac{5x^2 + 5}{x}$

(3) $f(x) = \frac{-3x^2 - 12}{x}$

(4) $f(x) = \frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2 + 4$

(5) $f(x) = \frac{3x^2 - 5x + 27}{x}$

(6) $f(x) = \frac{x^2 - 2x + 9}{2 - x}$

(7) $f(x) = \frac{1}{2}x^4 + 2x^3 + 2$

(8) $f(x) = \frac{4x^2 + 9x + 9}{x + 1}$

(9) $f(x) = \frac{2x^3 - 4}{x}$

(10) $f(x) = -\frac{1}{3}x^3 - \frac{1}{2}x^2 + 6x + 3$

(11) $f(x) = -\frac{6x^2 + 24}{x}$

(12) $f(x) = \frac{-5x^2 + 2x + 8}{x^2}$

(13) $f(x) = \frac{1}{4}x^4 - \frac{5}{3}x^3 + 2x^2 + 3$

(14) $f(x) = \frac{-2x^2 + 3x - 8}{x}$

(15) $f(x) = \frac{x^2 - x + 4}{x - 1}$

(16) $f(x) = \frac{3}{4}x^4 - 3x^3 + 4$

(17) $f(x) = \frac{2x^2 + 7x + 8}{x + 2}$

(18) $f(x) = \frac{3x^3 + 6}{x}$

(19) $f(x) = \frac{x^3}{x + 2}$

Find the absolute extrema of the function on the given interval.

(20) $f(x) = \frac{1}{2}x^4 - 4x^2 + 5;\ [1, 3]$

(21) $f(x) = \frac{-x^3 - 4}{x^2};\ [1, 4]$

(22) $f(x) = \frac{5}{2}x^4 - \frac{20}{3}x^3 + 6;\ [-1, 3]$

(23) $f(x) = \frac{3}{2}x^4 - 4x^3 + 4;\ [0, 3]$

(24) $f(x) = 2x^4 - 36x^2 + 20;\ [-4, -1]$

(25) $f(x) = \frac{2x^3 + 27}{2x^2};\ [2, 5]$

(26) $f(x) = \frac{40}{3}x^3 - 2x^4 + 10;\ [-1, 6]$

(27) $f(x) = -\frac{4}{5}x^5 + \frac{1}{2}x^4 + 8;\ [-2, 1]$

(28) $f(x) = \frac{x^2 + 25}{4x};\ [2, 6]$

(29) $f(x) = x^4 - 8x^2;\ [-2, 3]$

ANSWERS:

(1) Inc: $(-1, 3)$
Dec: $(-\infty, -1), (3, \infty)$
Local max: $(3, 13)$
Local min: $(-1, 7/3)$

(2) Inc: $(-\infty, -1), (1, \infty)$
Dec: $(-1, 0), (0, 1)$
Local max: $(-1, -10)$
Local min: $(1, 10)$

(3) Inc: $(-2, 0), (0, 2)$
Dec: $(-\infty, -2), (2, \infty)$
Local max: $(2, -12)$
Local min: $(-2, 12)$

(4) Inc: $(-2, 0), (1, \infty)$
Dec: $(-\infty, -2), (0, 1)$
Local max: $(0, 4)$
Local min: $(-2, 4/3), (1, 43/12)$

(5) Inc: $(-\infty, -3), (3, \infty)$
Dec: $(-3, 0), (0, 3)$
Local max: $(-3, -23)$
Local min: $(3, 13)$

(6) Inc: $(-1, 2), (2, 5)$
Dec: $(-\infty, -1), (5, \infty)$
Local max: $(5, -8)$
Local min: $(-1, 4)$

(7) Inc: $(-3, \infty)$
Dec: $(-\infty, -3)$
Local max: none
Local min: $(-3, -23/2)$

(8) Inc: $(-\infty, -2), (0, \infty)$
Dec: $(-2, -1), (-1, 0)$
Local max: $(-2, -7)$
Local min: $(0, 9)$

(9) Inc: $(-1, 0), (0, \infty)$
Dec: $(-\infty, -1)$
Local max: none
Local min: $(-1, 6)$

(10) Inc: $(-3, 2)$
Dec: $(-\infty, -3), (2, \infty)$
Local max: $(2, 31/3)$
Local min: $(-3, -21/2)$

(11) Inc: $(-2, 0), (0, 2)$
Dec: $(-\infty, -2), (2, \infty)$
Local max: $(2, -24)$
Local min: $(-2, 24)$

(12) Inc: $(-8, 0)$
Dec: $(-\infty, -8), (0, \infty)$
Local max: none
Local min: $(-8, -41/8)$

(13) Inc: $(0, 1), (4, \infty)$
Dec: $(-\infty, 0), (1, 4)$
Local max: $(1, 43/12)$
Local min: $(0, 3), (4, -23/3)$

(14) Inc: $(-2, 0), (0, 2)$
Dec: $(-\infty, -2), (2, \infty)$
Local max: $(2, -5)$
Local min: $(-2, 11)$

(15) Inc: $(-\infty, -1), (3, \infty)$
Dec: $(-1, 1), (1, 3)$
Local max: $(-1, -3)$
Local min: $(3, 5)$

(16) Inc: $(3, \infty)$
Dec: $(-\infty, 3)$
Local max: none
Local min: $(3, -65/4)$

(17) Inc: $(-\infty, -3), (-1, \infty)$
Dec: $(-3, -2), (-2, -1)$
Local max: $(-3, -5)$
Local min: $(-1, 3)$

(18) Inc: $(1, \infty)$
Dec: $(-\infty, 0), (0, 1)$
Local max: none
Local min: $(1, 9)$

(19) Inc: $(-3, -2), (-2, \infty)$
Dec: $(-\infty, -3)$
Local max: none
Local min: $(-3, 27)$

(20) Abs. min: $(2, -3)$
Abs. max: $(3, 19/2)$

(21) Abs. min: $(1, -5)$
Abs. max: $(2, -3)$

(22) Abs. min: $(2, -22/3)$
Abs. max: $(3, 57/2)$

(23) Abs. min: $(2, -4)$
Abs. max: $(3, 35/2)$

(24) Abs. min: $(-3. - 142)$
Abs. max: $(-1, -14)$

(25) Abs. min: $(3, 9/2)$
Abs. max: $(5, 277/50)$

(26) Abs. min:$(-1, -16/3)$
Abs. max: $(5, 1280/3)$

(27) Abs. min: $(1, 77/10)$
Abs. max: $(-2, 208/5)$

(28) Abs. min: $(5, 5/2)$
Abs. max: $(2, 29/8)$

(29) Abs. min: $(-2, -16)$, $(2, -16)$
Abs. max: $(3, 9)$