Elasticity of Demand Exercises

Problems

1. Suppose the demand curve for oPads is given by

$$p = \frac{500 - x}{10}.$$

   (a) Compute the elasticity of this demand function.

   (b) What is the price elasticity of demand when the price is \$30?

   (c) What is the percent change in the demand if the price is \$30 and increases by 4.5%?

2. Benson just opened a business selling calculators. The demand function for calculators can be given by $x = 400 - 2p^2$. Find the price for which he should sell the calculators in order to maximize revenue.

3. The demand for box seat tickets to watch the Habs can be described by the function

$$p = \left(100 - \frac{x}{10}\right)^2.$$

   Find the price elasticity of demand and determine whether management should increase or decrease the current ticket price of \$100 in order to increase revenue.

4. The current toll for the use of highway is \$250. Drivers use this highway because of its convenience even though there are other routes that are free. The provincial government does a study that determines that a toll of p dollars means x cars will use the road, where

$$p = -2\ln\left(\frac{x}{60000}\right)$$

   Compute the elasticity η at $p = 2.50$ and use it to determine whether an increase in the toll will increase or decrease revenue.

5. Currently 1800 people ride a commuter passenger ferry each day and pay \$4 for a ticket. The number of people x willing to ride the ferry at price p is determined by the relationship

$$p = \left(\frac{x - 3000}{600}\right)^2.$$

   The company would like to increase its revenue. Use the price elasticity of demand η to give advice to management on whether it should increase or decrease its price per passenger.

6. A cell phone supplier has determined that demand for its newest cell phone model is

$$xp + 30p + 50x = 15840,$$

   where p is the price (in dollars per phone) at which the supplier will be able to sell x cell phones.

   (a) Find the function that describes elasticity of demand for this product.

(b) If the current price is \$150 per phone, will revenue increase or decrease if the price is lowered slightly?

(c) What price should the cell phone supplier set for this cell phone to maximize its revenue from sales of the phone?

7. *A certain commodity satisfies the demand equation relating price p, and quantity demanded, x,

$$x = \frac{1000}{p^2}.$$

If the price of this commodity is lowered, will the revenue generated by its sales increase?

8. The price p (in dollars) and the demand x for a product are related by

$$p^2 + 2x^2 = 1100.$$

If the current price per unit is \$30, will revenue increase or decrease if the price is raised slightly?

Use the price elasticity of demand to solve this problem.

9. Shark Inc. has determined that demand for its newest netbook model is

$$\ln(x) - 2\ln(p) + 0.02p = 7,$$

where p is the price (in dollars per netbook) at which Shark will be able to sell x netbooks. Shark has determined that this model is valid for prices $p \geq 100$.

(a) If the current price is \$200 per unit, will revenue increase or decrease if the price is lowered slightly?

(b) Find the price that maximizes the revenue from sales of this netbook model.

Solutions

1. (a) $p' = \dfrac{-1}{10}$, so $\eta = \frac{p}{xp'} = \dfrac{\frac{500-x}{10}}{\frac{-x}{10}} = \dfrac{500-x}{10} \cdot \dfrac{10}{-x} = \dfrac{x-500}{x}$

   (b) When price is \$30, solving for x in the demand function gives $x = 200$. Then $\eta(200) = \frac{200-500}{200} = -1.5$.

   (c) $\eta = \dfrac{\text{\% Change in Demand}}{\text{\% Change in Price}} \Rightarrow -1.5 = \dfrac{\text{\% Change in Demand}}{4.5} \Rightarrow$
   % Change in Demand $= (-1.5)(4.5) = -6.75$. So the demand will decrease by -6.75%..

2. Differentiating implicitly,

$$\begin{aligned} \frac{d}{dx}(x) &= \frac{d}{dx}(400 - 2p^2) \\ 1 &= -4pp' \\ p' &= \frac{-1}{4p}. \end{aligned}$$

   Now $\eta = \dfrac{p}{\frac{-x}{4p}} = \dfrac{-4p^2}{x}$.

   Setting $\eta = -1$, we have $-1 = \dfrac{-4p^2}{x} \Rightarrow x = 4p^2$.

   Plug into the demand equation: $4p^2 = 400 - 2p^2 \Rightarrow 6p^2 = 400 \Rightarrow p = \sqrt{\dfrac{200}{3}}, \cancel{-\sqrt{\dfrac{200}{3}}}$
   $\Rightarrow p \approx \$8.16$.

3. $p' = \dfrac{-1}{5}(100 - \frac{x}{10})$,

   $\eta(x) = \dfrac{(100 - \frac{x}{10})^2}{\frac{-x}{5}(100 - \frac{x}{10})} = \dfrac{x - 1000}{2x}$.

   Setting $p = 100$ in the demand equation gives $x = 900$. So when $p = 100$, elasticity of demand is $\eta(900) = -\frac{1}{18} > -1$, and price is **inelastic**. When price is inelastic, $R(x)$ is a decreasing function – revenue increases as x decreases. A decrease in x corresponds to an increase in price, so in order to increase revenue management should **raise the price** from \$100.

4. $p' = \frac{-2}{x} \Rightarrow \eta = \dfrac{p}{x\left(\frac{-2}{x}\right)} = -\frac{p}{2}$

   If $p = 2.5$, $\eta = -1.25 < -1$. Since price is elastic, $R(x)$ is increasing. An increase in price corresponds to a decrease in x. Since $R(x)$ is increasing, **revenue will decrease** as x decreases. So an increase in the toll will decrease revenue.

5. $p' = \dfrac{2(x - 3000)}{600^2} \Rightarrow \eta = \dfrac{x - 3000}{2x}$

   When $x = 1800$, $\eta(1800) = -\frac{1}{3} > -1$, so price is inelastic, and $R(x)$ is decreasing. Revenue increases as x decreases, which corresponds to an increase in price. In order to increase revenue, management should **increase** the price per passenger.

6. (a) Differentiating implicitly with respect to x, we have

$$\begin{aligned} p + xp' + 30p' + 50 &= 0 \\ xp' + 30p' &= -50 - p \\ p' &= -\frac{p+50}{x+30} \Rightarrow \text{ So } \eta = \frac{p}{xp'} = \frac{-p(x+30)}{x(p+50)}. \end{aligned}$$

(b) If $p = 150$, we have

$$\begin{aligned} 150x + 30(150) + 50x &= 8500 \\ 200x &= 4000 \\ x &= 20 \end{aligned}$$

So elasticity is equal to $\eta = \dfrac{-150(20+30)}{20(150+50)} = -\frac{15}{8} < -1$. Demand is elastic, so $R(x)$ is increasing. If price is lowered slightly, revenue will increase.

(c) To maximize revenue, we set $\eta = -1$:

$$\begin{aligned} -1 &= \frac{-p(x+30)}{x(p+50)} \\ x(p+50) &= p(x+30) \\ xp + 50x &= xp + 30p \\ x &= \frac{3p}{5} \end{aligned}$$

Replace into the demand equation:

$$\begin{aligned} \frac{3p}{5} \cdot p + 30p + 50 \cdot \frac{3p}{5} &= 15840 \\ \frac{3}{5}p^2 + 60p - 8500 &= 0 \\ 3p^2 + 300p - 42500 = 0 &\Rightarrow p = 79.10, \cancel{-179.10} \end{aligned}$$

The price should be set to \$79.10 in order to maximize revenue.

7.

$$\frac{d}{dx}(x) = \frac{d}{dx}\left(\frac{1000}{p^2}\right) \Rightarrow 1 = \frac{-2000}{p^3}p' \Rightarrow p' = \frac{-p^3}{2000}$$

So $\eta = \frac{p}{xp'} = \frac{p}{\frac{1000}{p^2} \cdot \frac{-p^3}{2000}} = -2$.

Demand is price elastic, so $R(x)$ is an increasing function. If price is lowered, x will increase, so revenue will increase.

8.

$$2pp' + 4x = 0 \Rightarrow p' = \frac{-2x}{p}$$

$$\eta = \frac{p}{xp'} = \frac{p}{x}\frac{p}{-2x} = \frac{-p^2}{2x^2} = \frac{-p^2}{1100 - p^2}$$

$\eta(30) = -\frac{30^2}{1100-30^2} = -\frac{900}{200} = -4.5$

Demand is price elastic, so $R(x)$ is increasing. If price is raised, x decreases, so revenue will decrease.

9.

$$\frac{1}{x} - \frac{2p'}{p} + 0.02p' = 0 \Rightarrow p' = \frac{50p}{100x - xp}$$

$$\eta = \frac{p}{xp'} = \frac{p}{x}\frac{x(100-p)}{50p} = \frac{100-p}{50}$$

(a) $p = 200 \Rightarrow \eta = \frac{100-200}{50} = -2.$
Revenue will increase as price decreases, since demand is price elastic.

(b) $-1 = \frac{100-p}{50} \Rightarrow -50 = 100 - p \Rightarrow p = 150$
Revenue is maximized when $p = \$150$.