

Elasticity of Demand

The **price elasticity of demand** (which is often shortened to *demand elasticity*) is defined to be the relative rate of change in quantity demanded, x , divided by the relative rate of change in price, p . If we write everything in terms of quantity (by using the demand equation $p = p(x)$), we get:

$$\text{Relative rate of change of quantity: } \frac{\frac{d}{dx}(x)}{x} = \frac{1}{x}$$

$$\text{Relative rate of change of price: } \frac{\frac{d}{dx}(p(x))}{p(x)} = \frac{p'(x)}{p(x)}$$

Using these definitions, the formula for the demand elasticity (η) is then:

$$\eta = \frac{p(x)}{xp'(x)}.$$

Note that the law of demand implies that $\frac{dp}{dx} < 0$, and so η will be a negative number.

Why do we care about demand elasticities? One way to see why price elasticity of demand might be useful is to consider the question: How will revenue change as we change the price?

Recall that revenue is price times quantity demanded ($R = px$). With x as our variable, we have:

$$R(x) = p(x) \cdot x$$

. Now the derivative of a function tells us how that function will change: If $R'(x) > 0$ then the revenue is increasing at that price point, and $R'(x) < 0$ would say that revenue is decreasing at that price point. So, we compute $R'(x)$ using the general revenue equation:

$$\begin{aligned} R'(x) &= xp'(x) + p(x) \\ &= xp'(x) \left[1 + \frac{p(x)}{xp'(x)} \right] \end{aligned}$$

Note that x is always positive, and $p'(x)$ is always negative, so the sign of $R'(x)$ depends solely on the quantity in the brackets. We note that whether this quantity is positive or negative is determined by the size of the second term, $p(x)/(xp'(x))$, which is precisely the demand elasticity η as we have defined it. Thus, the size in absolute value, of η relative to 1 will determine how the revenue changes as the price changes. (This is why some economists prefer to use the positive version of demand elasticity).

Now, suppose a manager wants to find out how consumers will react when he increases the price of a good. If increasing the price increases revenue, the manager would have made a good decision. If increasing the price decreases revenue, the manager would have made a bad decision. We use elasticities to see how sensitive consumers are to price changes.

Note that the negative sign of the demand elasticity as we have defined it encodes how demand responds to price changes: *As price increases, quantity demanded decreases, and as price decreases, quantity demanded increases.* That is, the fact that η is negative tells us price p and quantity de-

manded x move in opposite directions!

Remember, the negative values of η comes from the factor $p'(x)$, which is the slope of the demand curve and is negative by the law of demand. (If you have a good for which the elasticity is not negative, this good does not follow the law of demand.) Economists use the following terminology:

Case 1. When $\eta < -1$, we say the good is price **elastic**.

In this case, (% Change in Quantity) > (% Change in Price), and so, for a 1% change in price, there is a greater than 1% change in quantity demanded.

When the good is price elastic, $R'(x) = xp'(x) \left[1 + \frac{p(x)}{xp'(x)} \right]$ is positive, so $R(x)$ is an increasing function. Revenue will increase when x increases. By the law of demand, this corresponds to a decrease in price. Management should **decrease the price** to have a higher revenue.

Case 2. When $\eta > -1$, we say the good is price **inelastic**.

In this case, (% Change in Quantity) < (% Change in Price), and so, for a 1% change in price, there is a less than 1% change in quantity demanded.

When the good is price inelastic, $R'(x) = xp'(x) \left[1 + \frac{p(x)}{xp'(x)} \right]$ is negative, so $R(x)$ is a decreasing function. Revenue will increase when x decreases. By the law of demand, this corresponds to an increase in price. Management should **increase the price** to have a higher revenue.

Case 3. When $\eta = -1$, we say the good price is **unit elastic**.

In this case, (%Change in Quantity) = (%Change in Price), and so, for a 1% change in price, there is also a 1% change in quantity demanded.

This is the optimal price which means it maximizes revenue. (Why is this so? Draw a graph of revenue versus price for the case of linear demand curve to see what is happening).

According to economic theory, the primary determinant of the price elasticity of demand is the availability of substitutes. If many substitute goods are available, then demand is *price elastic*. For example, if the price of Pepsi increases, consumers can easily switch to Coke. So the price elasticity of Pepsi is very high. Whereas, if there are few substitutes, then demand is *price inelastic*. Even if the price of a basic food item increases, one has to buy this food item to survive. For example, in much of the world, rice is a staple food item, and is an item which is highly price inelastic.

There are other types of elasticities besides price elasticity of demand, but we will not consider them in this course.

Optional Remarks

As stated earlier, for our purposes, we will be considering only goods which follow the law of demand, i.e. goods where a price increase correlates to a **decrease** in quantity of demand. For the sake of completeness, there are in fact types of goods that do not satisfy this law, for instance **Giffen goods** and **Veblen goods**.

Giffen goods are inferior goods with no reasonable substitute that constitute a substantial percentage of the buyer's income.

As Mr. Giffen has pointed out, a rise in the price of bread makes so large a drain on the resources of the poorer labouring families and raises the marginal utility of money to them so much that they are forced to curtail their consumption of meat and the more expensive farinaceous foods: and, bread being still the cheapest food which they can get and will take, they consume more, and not less of it.

—Alfred Marshall, *Principles of Economics* (1895 ed.)

Veblen goods (also known as positional goods) are commodities whose demand is proportional to their price. The idea is that some luxury goods become more attractive as price increases since this increases the perception of high status and exclusivity associated to them. On the other hand, a drop in price would decrease demand as they may lose their allure once they are no longer seen as exclusive items. The examples usually brought up include designer handbags and shoes, luxury cars, and high-end wines.