

201-103-RE - Supplement E: Applications of the First Derivative

Find the intervals of increase and decrease, and and local extrema of the following functions:

- (1) $f(x) = -\frac{x^3}{3} + x^2 + 3x + 4$ (2) $f(x) = \frac{5x^2 + 5}{x}$ (3) $f(x) = \frac{-3x^2 - 12}{x}$
- (4) $f(x) = \frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2 + 4$ (5) $f(x) = \frac{3x^2 - 5x + 27}{x}$ (6) $f(x) = \frac{x^2 - 2x + 9}{2 - x}$
- (7) $f(x) = \frac{1}{2}x^4 + 2x^3 + 2$ (8) $f(x) = \frac{4x^2 + 9x + 9}{x + 1}$ (9) $f(x) = \frac{2x^3 - 4}{x}$
- (10) $f(x) = -\frac{1}{3}x^3 - \frac{1}{2}x^2 + 6x + 3$ (11) $f(x) = -\frac{6x^2 + 24}{x}$ (12) $f(x) = \frac{-5x^2 + 2x + 8}{x^2}$
- (13) $f(x) = \frac{1}{4}x^4 - \frac{5}{3}x^3 + 2x^2 + 3$ (14) $f(x) = \frac{-2x^2 + 3x - 8}{x}$ (15) $f(x) = \frac{x^2 - x + 4}{x - 1}$
- (16) $f(x) = \frac{3}{4}x^4 - 3x^3 + 4$ (17) $f(x) = \frac{2x^2 + 7x + 8}{x + 2}$ (18) $f(x) = \frac{3x^3 + 6}{x}$
- (19) $f(x) = \frac{x^3}{x + 2}$

Find the absolute extrema of the function on the given interval.

- (20) $f(x) = \frac{1}{2}x^4 - 4x^2 + 5; [1, 3]$ (21) $f(x) = \frac{-x^3 - 4}{x^2}; [1, 4]$ (22) $f(x) = \frac{5}{2}x^4 - \frac{20}{3}x^3 + 6; [-1, 3]$
- (23) $f(x) = \frac{3}{2}x^4 - 4x^3 + 4; [0, 3]$ (24) $f(x) = 2x^4 - 36x^2 + 20; [-4, -1]$ (25) $f(x) = \frac{2x^3 + 27}{2x^2}; [2, 5]$
- (26) $f(x) = \frac{40}{3}x^3 - 2x^4 + 10; [-1, 6]$ (27) $f(x) = -\frac{4}{5}x^5 + \frac{1}{2}x^4 + 8; [-2, 1]$ (28) $f(x) = \frac{x^2 + 25}{4x}; [2, 6]$
- (29) $f(x) = x^4 - 8x^2; [-2, 3]$
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ANSWERS:

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| (1) Inc: $(-1, 3)$
Dec: $(-\infty, -1), (3, \infty)$
Local max: $(3, 13)$
Local min: $(-1, 7/3)$ | (2) Inc: $(-\infty, -1), (1, \infty)$
Dec: $(-1, 0), (0, 1)$
Local max: $(-1, -10)$
Local min: $(1, 10)$ | (3) Inc: $(-2, 0), (0, 2)$
Dec: $(-\infty, -2), (2, \infty)$
Local max: $(2, -12)$
Local min: $(-2, 12)$ |
| (4) Inc: $(-2, 0), (1, \infty)$
Dec: $(-\infty, -2), (0, 1)$
Local max: $(0, 4)$
Local min: $(-2, 4/3), (1, 43/12)$ | (5) Inc: $(-\infty, -3), (3, \infty)$
Dec: $(-3, 0), (0, 3)$
Local max: $(-3, -23)$
Local min: $(3, 13)$ | (6) Inc: $(-1, 2), (2, 5)$
Dec: $(-\infty, -1), (5, \infty)$
Local max: $(5, -8)$
Local min: $(-1, 4)$ |
| (7) Inc: $(-3, \infty)$
Dec: $(-\infty, -3)$
Local max: none
Local min: $(-3, -23/2)$ | (8) Inc: $(-\infty, -2), (0, \infty)$
Dec: $(-2, -1), (-1, 0)$
Local max: $(-2, -7)$
Local min: $(0, 9)$ | (9) Inc: $(1, \infty)$
Dec: $(-\infty, 0), (0, 1)$
Local max: none
Local min: $(1, -2)$ |
| (10) Inc: $(-3, 2)$
Dec: $(-\infty, -3), (2, \infty)$
Local max: $(2, 31/3)$
Local min: $(-3, -21/2)$ | (11) Inc: $(-2, 0), (0, 2)$
Dec: $(-\infty, -2), (2, \infty)$
Local max: $(2, -24)$
Local min: $(-2, 24)$ | (12) Inc: $(-8, 0)$
Dec: $(-\infty, -8), (0, \infty)$
Local max: none
Local min: $(-8, -41/3)$ |
| (13) Inc: $(0, 1), (4, \infty)$
Dec: $(-\infty, 0), (1, 4)$
Local max: $(1, 43/12)$
Local min: $(0, 3), (4, -23/3)$ | (14) Inc: $(-2, 0), (0, 2)$
Dec: $(-\infty, -2), (2, \infty)$
Local max: $(2, -5)$
Local min: $(-2, 11)$ | (15) Inc: $(-\infty, -1), (3, \infty)$
Dec: $(-1, 1), (1, 3)$
Local max: $(-1, -3)$
Local min: $(3, 5)$ |
| (16) Inc: $(3, \infty)$
Dec: $(-\infty, 3)$
Local max: none
Local min: $(3, -65/4)$ | (17) Inc: $(-\infty, -3), (-1, \infty)$
Dec: $(-3, -2), (-2, -1)$
Local max: $(-3, -47)$
Local min: $(-1, 3)$ | (18) Inc: $(1, \infty)$
Dec: $(-\infty, 0), (0, 1)$
Local max: none
Local min: $(1, 9)$ |
| (19) Inc: $(-4, -2), (-2, \infty)$
Dec: $(-\infty, -4)$
Local max: none
Local min: $(-4, 32)$ | (20) Abs. min: $(2, -3)$
Abs. max: $(3, 19/2)$ | (21) Abs. min: $(1, -5)$
Abs. max: $(2, -3)$ |
| (22) Abs. min: $(2, -22/3)$
Abs. max: $(3, 57/2)$ | (23) Abs. min: $(2, -4)$
Abs. max: $(3, 35/2)$ | (24) Abs. min: $(-3, -142)$
Abs. max: $(-1, -14)$ |
| (25) Abs. min: $(3, 9/2)$
Abs. max: $(5, 277/50)$ | (26) Abs. min: $(-1, -16/3)$
Abs. max: $(5, 1280/3)$ | (27) Abs. min: $(1, 77/10)$
Abs. max: $(-2, 208/5)$ |
| (28) Abs. min: $(5, 5/2)$
Abs. max: $(2, 29/8)$ | (29) Abs. min: $(-2, -16), (2, -16)$
Abs. max: $(3, 9)$ | |