Math NYC: Test 1 (June 19, 2006)   Name: ______________________________

[marks]

Answer the questions on looseleaf. Show all your work! Scientific (non-graphing) calculators are permitted. Use proper mathematical notation and clearly indicate your final answer. Hand in the question sheet along with your answers.

Note: The test will be marked out of 40. (44 marks are available.)

[8] 1. Find the general solution, using Gaussian or Gauss-Jordan elimination.

$$\begin{aligned} 4x_1 + 2x_2 \phantom{{}+3x_3} - 2x_4 &= 4 \\ -x_1 - 2x_2 + 3x_3 + 5x_4 &= 2 \\ 3x_1 \phantom{{}-2x_2} + x_3 + 2x_4 &= -2 \end{aligned}$$

[4] 2. Given $A = \begin{bmatrix} 2 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 0 \\ 1 & 2 \\ 5 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$, find D such that $AB - 4D = C^2$.

[6] 3. Find the inverse of the matrix: $\begin{bmatrix} 1 & 3 & 2 \\ -1 & -4 & -3 \\ 3 & 7 & 5 \end{bmatrix}$

[3] 4. Given that $\begin{bmatrix} 1 & 4 & 2 \\ 2 & 7 & 3 \\ -2 & -7 & -2 \end{bmatrix}^{-1} = \begin{bmatrix} -7 & 6 & 2 \\ 2 & -2 & -1 \\ 0 & 1 & 1 \end{bmatrix}$,

use this information to solve the system of linear equations (no other method will be accepted):

$$\begin{aligned} x_1 + 4x_2 + 2x_3 &= 5 \\ 2x_1 + 7x_2 + 3x_3 &= -6 \\ -2x_1 - 7x_2 - 2x_3 &= 1 \end{aligned}$$

Flip the page for more questions!

[5] 5. Given the system of linear equations:

$$\begin{aligned} x + y - z &= 0 \\ x + (k+1)y + 2z &= 0 \\ x + y + (k-5)z &= 0 \end{aligned}$$

find all values of k (if any) for which the system has:

(a) no solutions

(b) one solution

(c) infinitely many solutions

[8] 6. Find the determinant of the following matrix:

$$\begin{bmatrix} 6 & 3 & 15 & 2 \\ 4 & 2 & 10 & -5 \\ -2 & 1 & 0 & 1 \\ 3 & -1 & 0 & 4 \end{bmatrix}$$

[6] 7. Indicate whether the statement is true or not. Justify your answer with a logical argument or a counterexample. (A, B, C, and D are matrices.)

(a) $(AB)^{-1} = B^{-1}A^{-1}$

(b) If $AB + AC = DC + DB$ then $A = D$.

(c) If A is symmetric then so is $2A + 2A^{\mathrm{T}}$.

[4] 8. Express $A = \begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix}$ as a product of elementary matrices.