Solutions for Quiz 1 (Rewrite)

1.

$$\begin{bmatrix} 2 & 4 & 0 & 2 & -8 \\ -1 & -4 & -2 & 3 & 5 \\ 3 & 4 & -2 & 7 & -11 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -2 & 5 & -3 \\ 0 & 1 & 1 & -2 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

So $x_1 = -3 + 2s - 5t$, $x_2 = -\frac{1}{2} - s + 2t$, $x_3 = t$, and $x_4 = s$.

2. (a) $(M)_{34} = \begin{bmatrix} 0 & 6 \end{bmatrix} \begin{bmatrix} 4 \\ 2 \end{bmatrix} = 12$

   (b) $\operatorname{tr}(AA^{\mathrm{T}}) = 2^2 + 3^2 + (-1)^2 + 5^2 + 0^2 + 6^2 = 75$

3. (a) FALSE. Many counterexamples are possible. For instance:

$$\begin{aligned} 2x + y &= 0 \\ x - y &= 0 \\ x + 2y &= 0 \end{aligned}$$

   (This example is a homogeneous system, therefore it must be consistent.)

   (b) TRUE. You can prove it by writing it out with general 2 x 2 matrices.

$$\begin{aligned} (A+B)^{\mathrm{T}} &= \begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{bmatrix}^{\mathrm{T}} \\ &= \begin{bmatrix} a_{11}+b_{11} & a_{21}+b_{21} \\ a_{12}+b_{12} & a_{22}+b_{22} \end{bmatrix} \\ &= \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \end{bmatrix} \\ &= A^{\mathrm{T}} + B^{\mathrm{T}} \end{aligned}$$

   (c) FALSE. Many counterexamples are possible. For instance:
   Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$. Then $(A+B)^{-1}$ doesn't even exist.