Extra answers (not included in textbook or solutions manual) for Chapter 1.3

3. a) $\begin{bmatrix} 7 & 6 & 5 \\ -2 & 1 & 3 \\ 7 & 3 & 7 \end{bmatrix}$ **b)** $\begin{bmatrix} -5 & 4 & -1 \\ 0 & -1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$ **c)** $\begin{bmatrix} 15 & 0 \\ -5 & 10 \\ 5 & 5 \end{bmatrix}$ **d)** $\begin{bmatrix} -7 & -28 & -14 \\ -21 & -7 & -35 \end{bmatrix}$

f) $\begin{bmatrix} 22 & -6 & 8 \\ -2 & 4 & 6 \\ 10 & 0 & 4 \end{bmatrix}$ **h)** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ **i)** 5 **k)** 168 **l)** undefined (A is not a square matrix).

6. c) $\begin{bmatrix} 2 & -10 & 11 \\ 13 & 2 & 5 \\ 4 & -3 & 13 \end{bmatrix}$ **e)** $\begin{bmatrix} 40 & 72 \\ 26 & 42 \end{bmatrix}$

14. a)

$$\begin{aligned} 3x_1 - x_2 + 2x_3 &= 2 \\ 4x_1 + 3x_2 + 7x_3 &= -1 \\ -2x_1 + x_2 + 5x_3 &= 4 \end{aligned}$$

b)

$$\begin{aligned} 3w - 2x + z &= 0 \\ 5w + 2y - 2z &= 0 \\ 3w + x + 4y + 7z &= 0 \\ -2w + 5x + y + 6z &= 0 \end{aligned}$$

18. b) Claim: If B has a column of zeros and AB is defined, then AB has a column of zeros.

Proof: The entry in the i^{th} row and j^{th} column of AB is

$$a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{in}b_{nj}$$

(by the definition of matrix multiplication). Assuming the j^{th} column of B consists entirely of zeros, each of the b_{*j}'s are equal to zero, so the above sum equals zero. Therefore every entry in the j^{th} column of AB must be zero.

19. First, suppose $A \neq 0$. Since $kA = 0$, the product of k and each entry of A must be 0; since some entries in A are nonzero, k must be 0.

Second, suppose $k \neq 0$. If A had any nonzero entry, the corresponding entry in kA would be nonzero. Therefore, A must be the zero matrix.

28. There are infinitely many possibilities.

Here are a few that work: $\begin{bmatrix} y & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} y & -z & y \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} y & 0 & 0 \\ -2yz & xz & xy \\ 0 & 0 & 0 \end{bmatrix}$.