

Solution: Webwork #2, problem #17  
(NYA Winter 2007 – Alice McLeod)

Webwork problem number 17 looked something like this (your numbers may be different):

$$\lim_{x \rightarrow 0} \frac{1 - \cos(5x)}{1 - \cos(13x)}$$

Here's a sample solution:

$$\begin{aligned} & \lim_{x \rightarrow 0} \left( \frac{1 - \cos(5x)}{1 - \cos(13x)} \cdot \frac{1 + \cos(5x)}{1 + \cos(5x)} \cdot \frac{1 + \cos(13x)}{1 + \cos(13x)} \right) \\ &= \lim_{x \rightarrow 0} \left( \frac{(1 - \cos(5x))(1 + \cos(5x))}{(1 - \cos(13x))(1 + \cos(13x))} \cdot \frac{1 + \cos(13x)}{1 + \cos(5x)} \right) \\ &= \lim_{x \rightarrow 0} \left( \frac{1 - \cos^2(5x)}{1 - \cos^2(13x)} \cdot \frac{1 + \cos(13x)}{1 + \cos(5x)} \right) \\ &= \lim_{x \rightarrow 0} \left( \frac{\sin^2(5x)}{\sin^2(13x)} \cdot \frac{1 + \cos(13x)}{1 + \cos(5x)} \right) \\ &= \lim_{x \rightarrow 0} \left( \frac{(\sin(5x))(\sin(5x))}{(\sin(13x))(\sin(13x))} \cdot \frac{1 + \cos(13x)}{1 + \cos(5x)} \right) \\ &= \lim_{x \rightarrow 0} \left( \frac{\sin(5x)}{5x} \cdot \frac{\sin(5x)}{5x} \cdot \frac{13x}{\sin(13x)} \cdot \frac{13x}{\sin(13x)} \cdot \frac{5^2}{13^2} \cdot \frac{1 + \cos(13x)}{1 + \cos(5x)} \right) \\ &= 1 \cdot 1 \cdot 1 \cdot 1 \cdot \frac{5^2}{13^2} \cdot \frac{1 + 1}{1 + 1} \\ &= \frac{5^2}{13^2} \end{aligned}$$